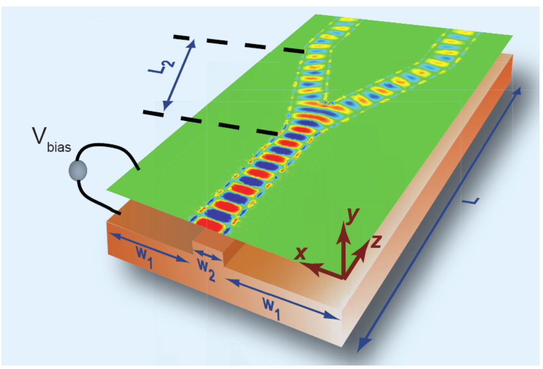
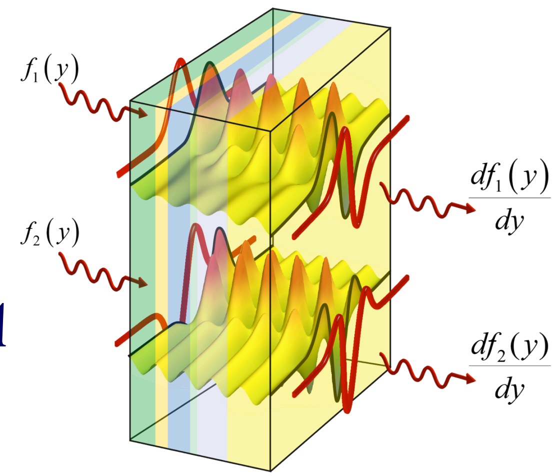
Fields and Waves in Metamaterials
Part 1



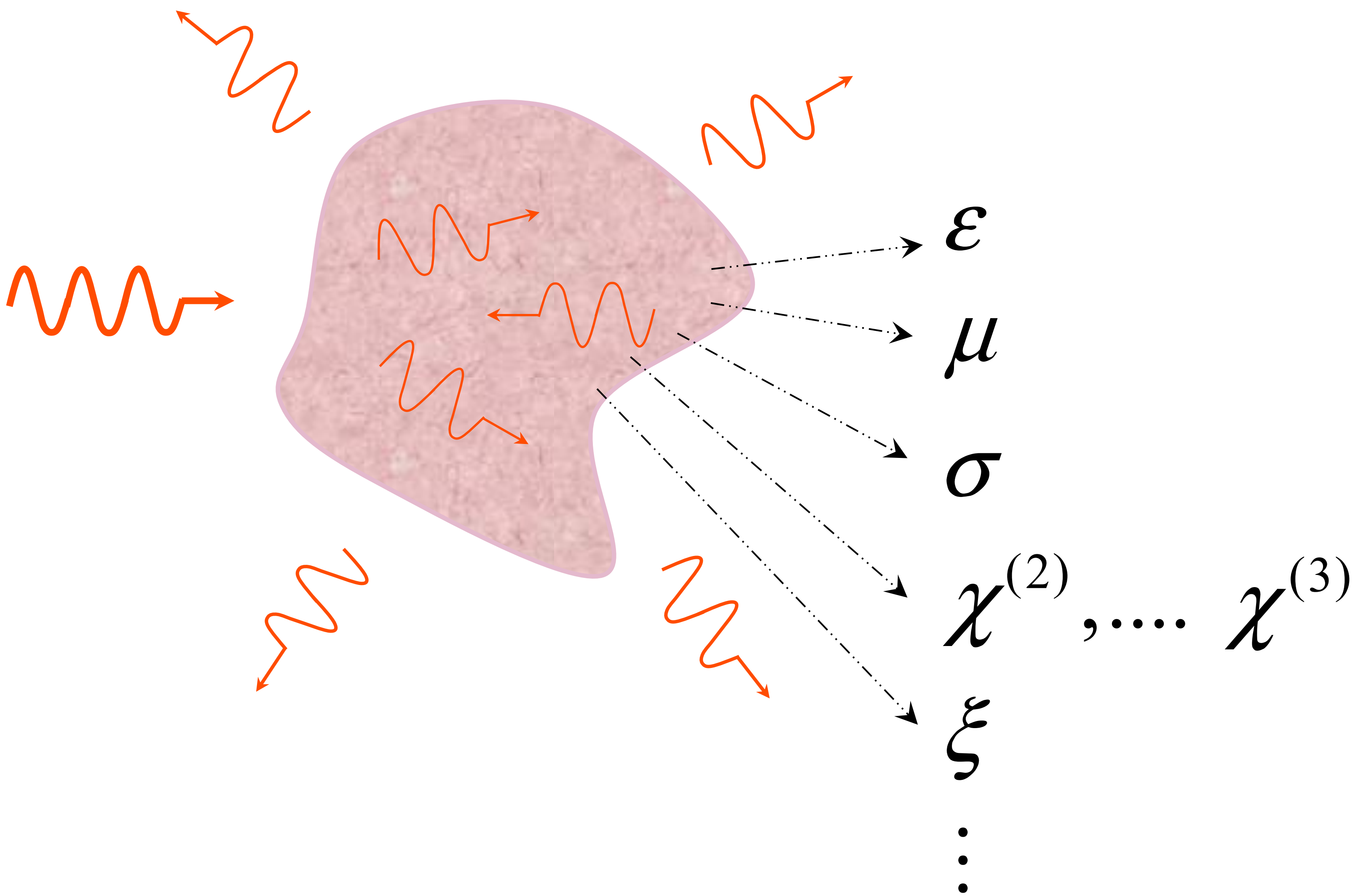
Nader Engheta

University of Pennsylvania
Philadelphia, PA 19104, USA



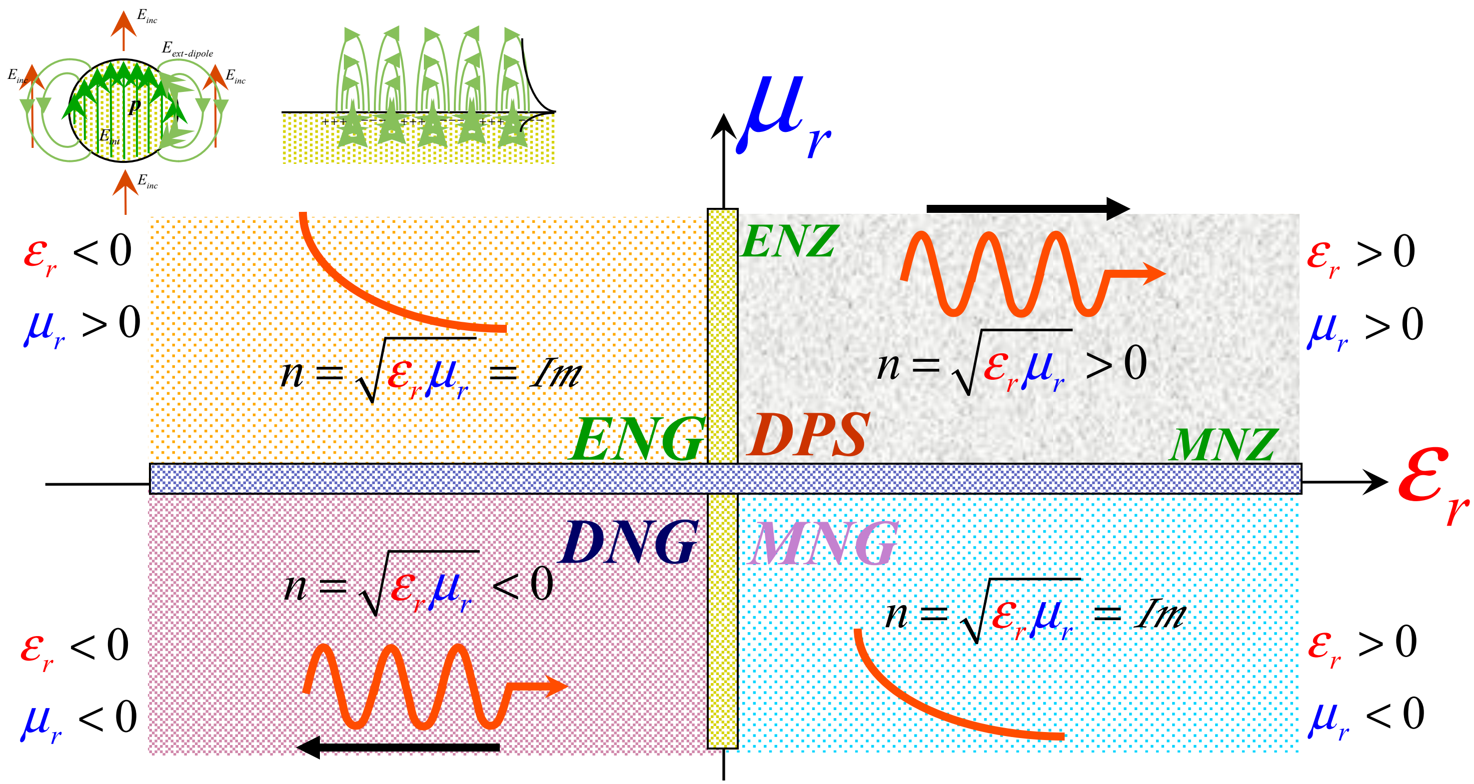
August 16-17, 2014

Light-Matter Interaction

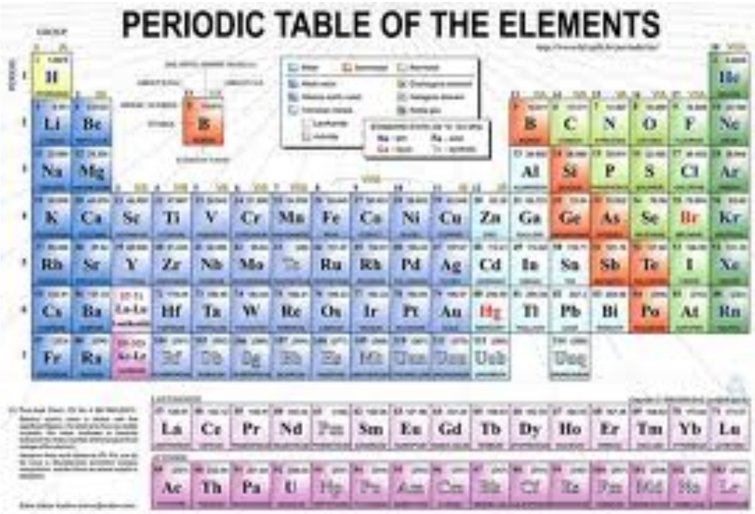


Metamaterials and Plasmonic Phenomena

$$\boldsymbol{E} = \boldsymbol{E_o} e^{ikz} e^{-i\omega t} \qquad k = \omega\sqrt{\varepsilon\mu} = \frac{2\pi}{\lambda} \qquad n = \sqrt{\varepsilon_r \mu_r}$$

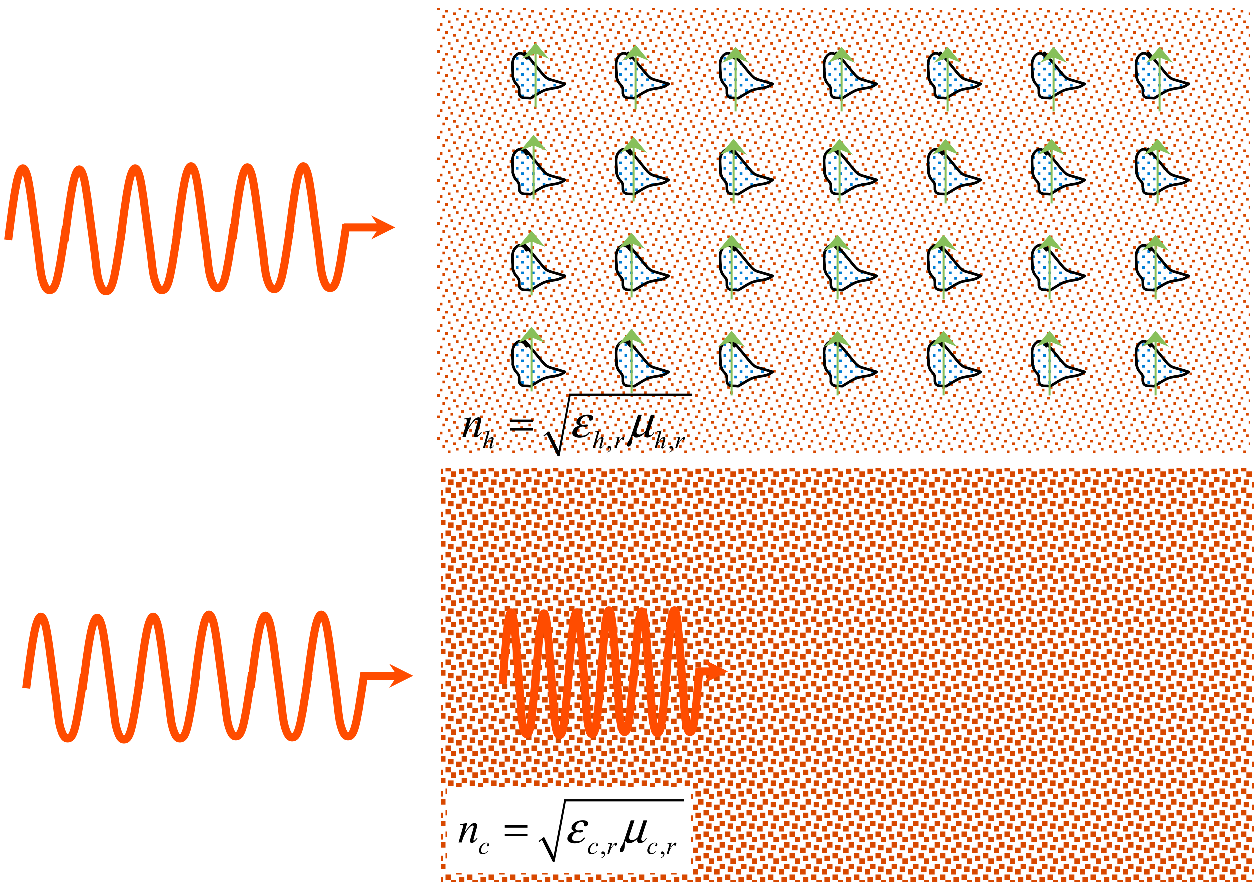


"Natural" Materials



"Artificially" Engineered Materials

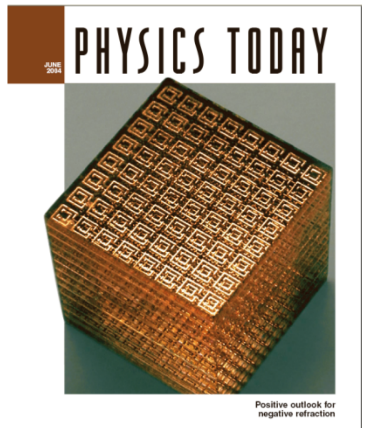
- *Particulate Composite Materials*



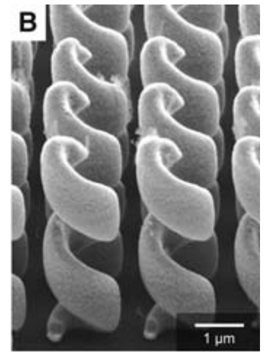
- *Composition*
- *Alignment*
- *Arrangement*
- *Density*
- *Host Medium*
- *Geometry/Shape*

Metamaterials Samples (2000-2013)

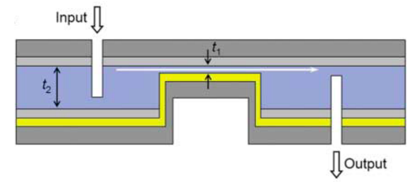
Smith, Schultz group (2000)



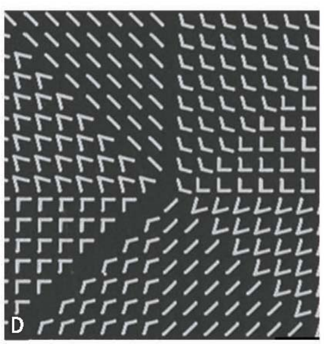
Boeing group



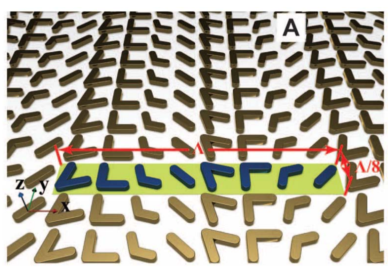
Wegener group (2009)



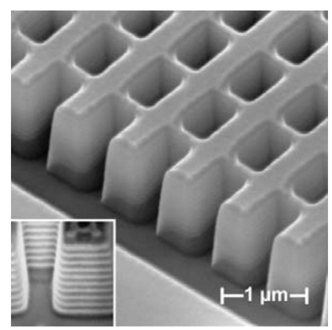
Atwater group (2007)



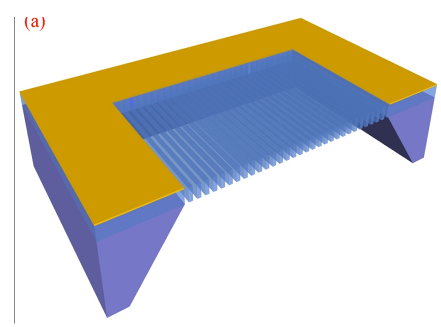
Capasso group (2011)



Shalaev group (2011)



Zhang group (2008)



Engheta group (2012)

Metamaterial Applications (2000-2013)

Cloaking

Perfect Lens

Ultrathin Cavities

Transformation Optics

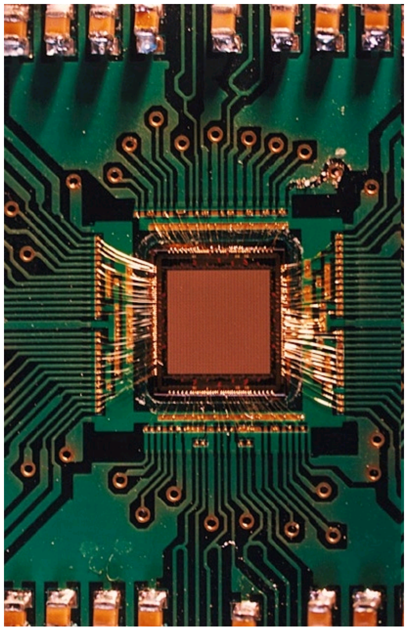
Hyperlens

ENZ & MNZ

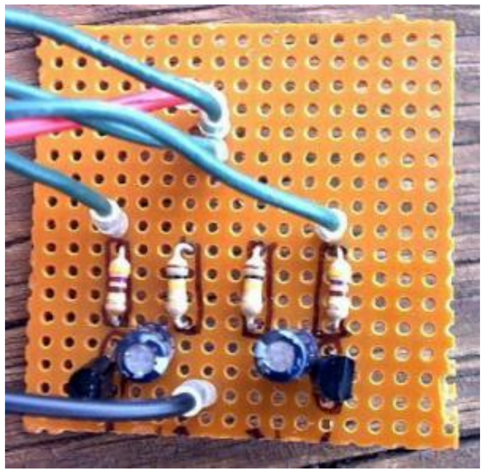
Metasurfaces

Metatronics

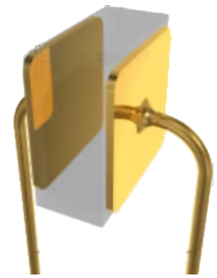
Electronic Modules



http://www.imrc.hw.ac.uk/New_versions/Home_files/Microelectronics.jpg

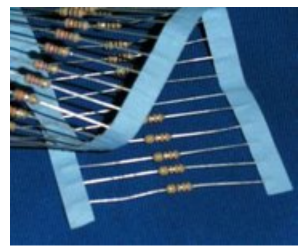


C

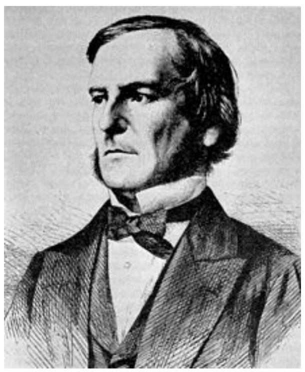


L

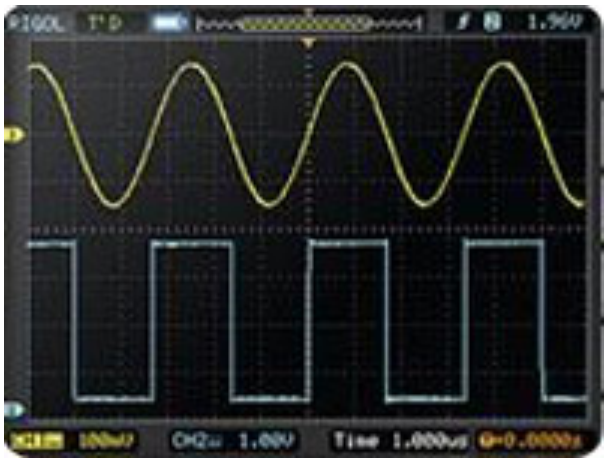
R

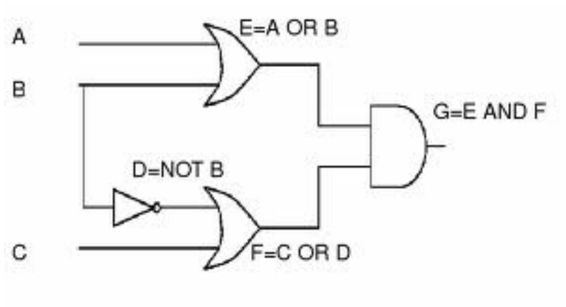


Analog vs Digital



George Boole

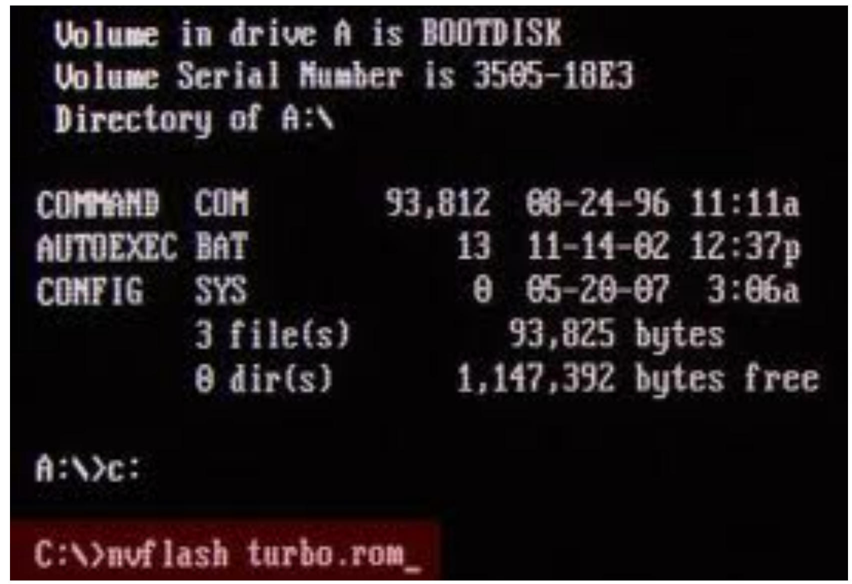




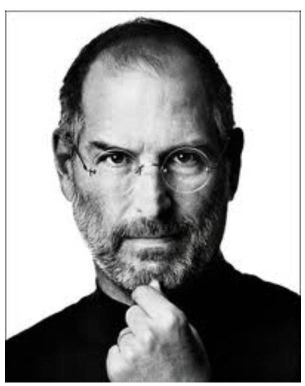
$$f(t) = \sin(\omega t)$$

$$u(t) = 1\,0\,0\,1\,1\,1\,0 \cdots$$

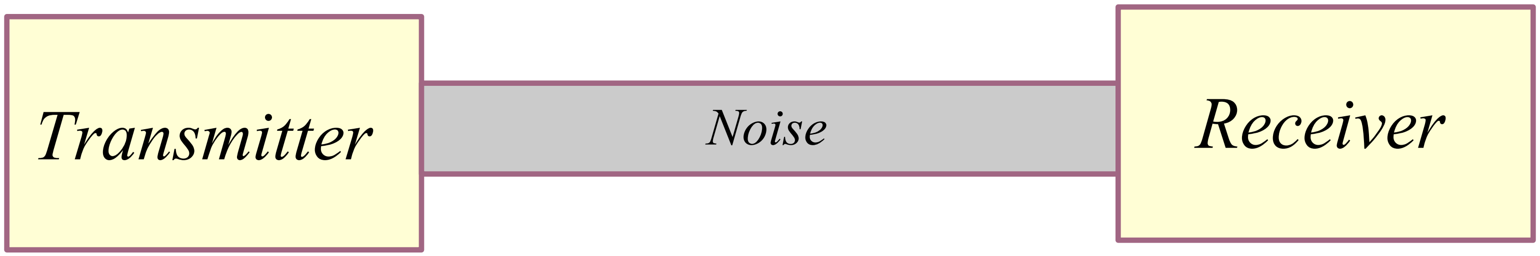
iPhone *DOS*

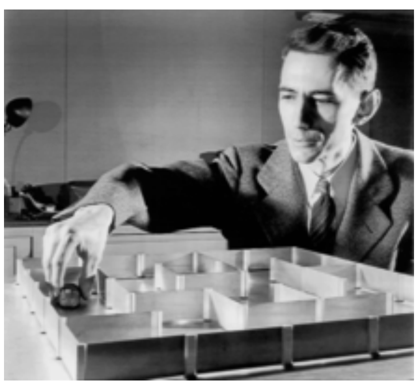


http://t0.gstatic.com/images?q=tbn:ANd9GcQ2jC_aCeZHKyjVou0Q_xOq0LG3FkyuW963_OLqcM07rld4EHAUsA



Claude Shannon & Channel Capacity



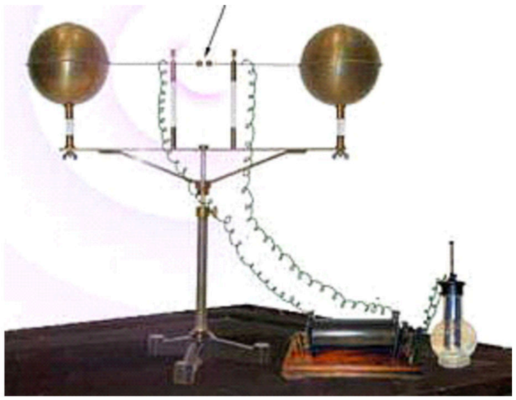


$$\mathit{Channel\ Capacity} = B \log_2\left(1 + \frac{S}{N}\right)$$

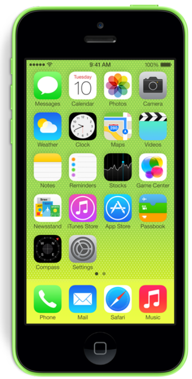
C. Shannon

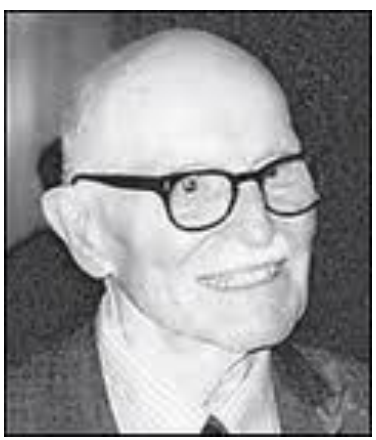
Development of Antennas

a)



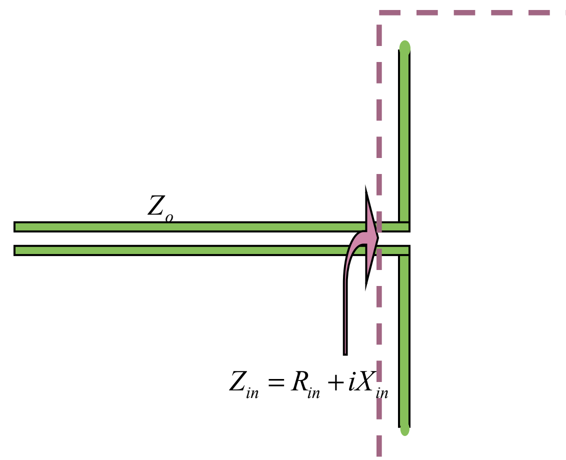
From: http://www.sparkmuseum.com



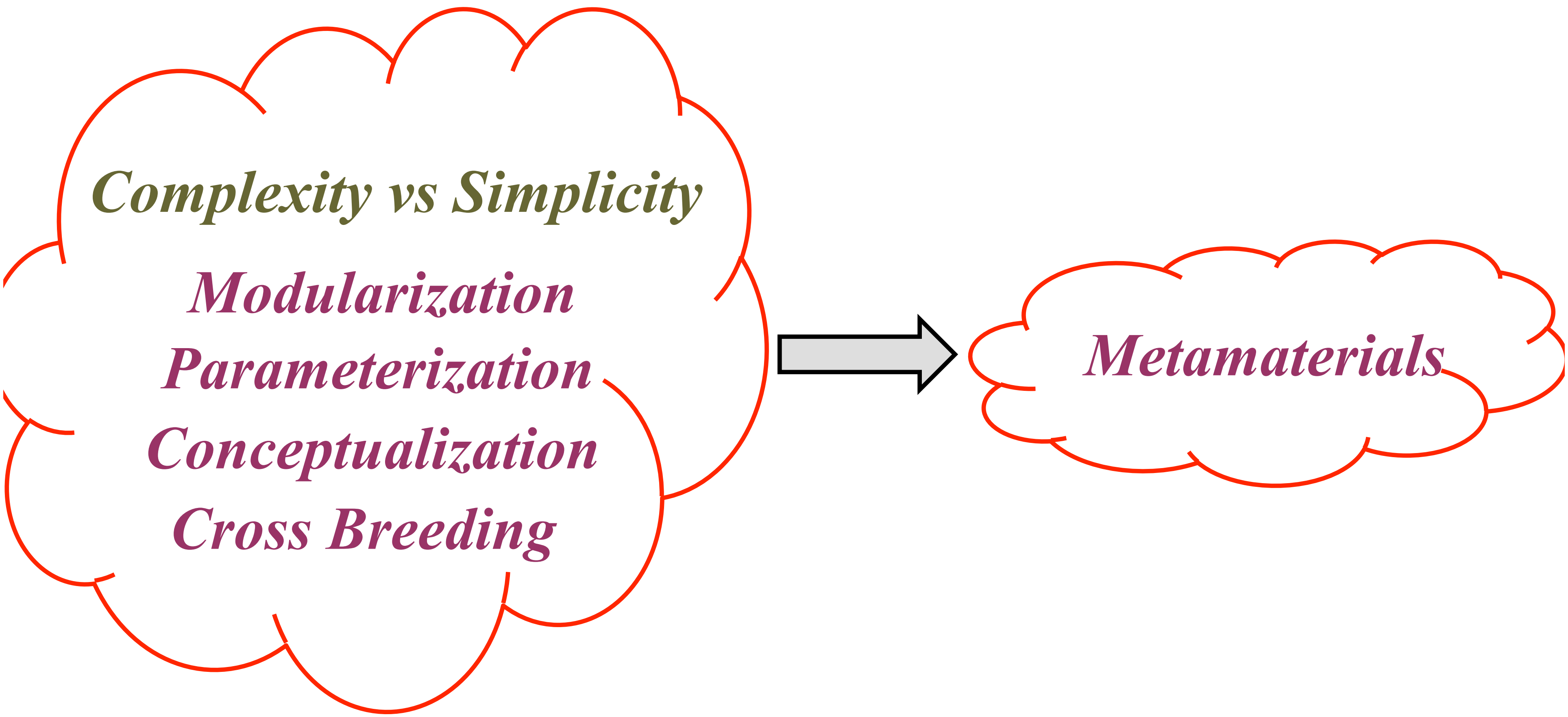


R. W. P. King

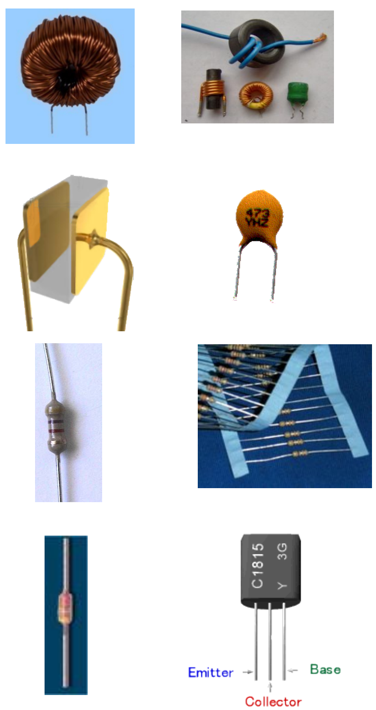
S. A. Schelkunoff



How about Metamaterials?



Metamaterial Gadgets?



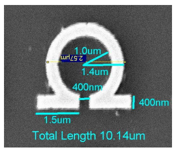
L

C

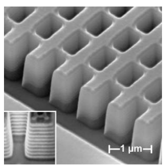
R

Tr

Metamaterial Gadgets?

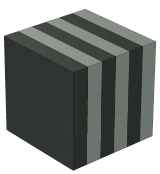


ε



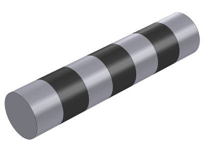
μ

σ



$\chi^{(2)}$

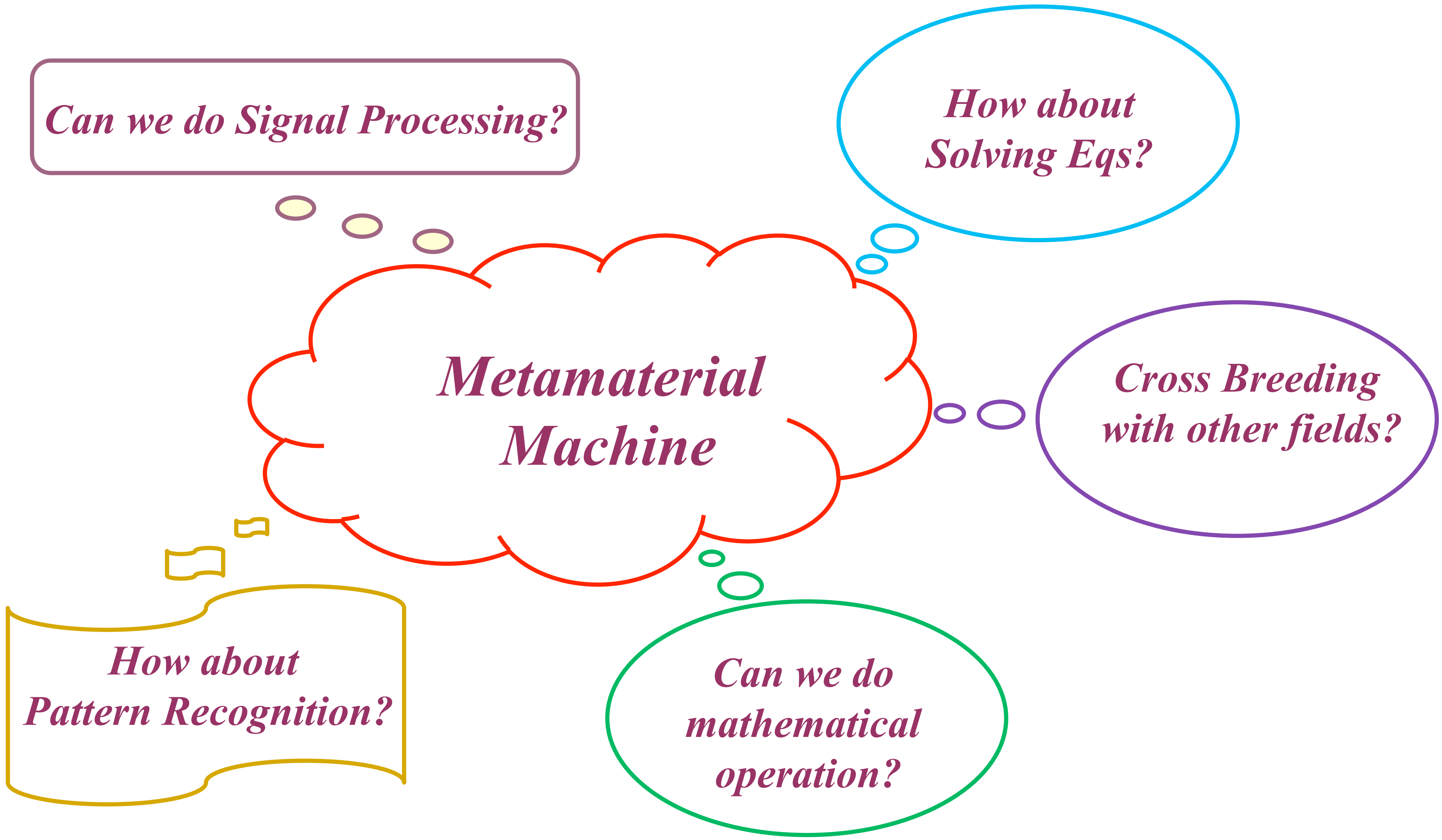
ξ



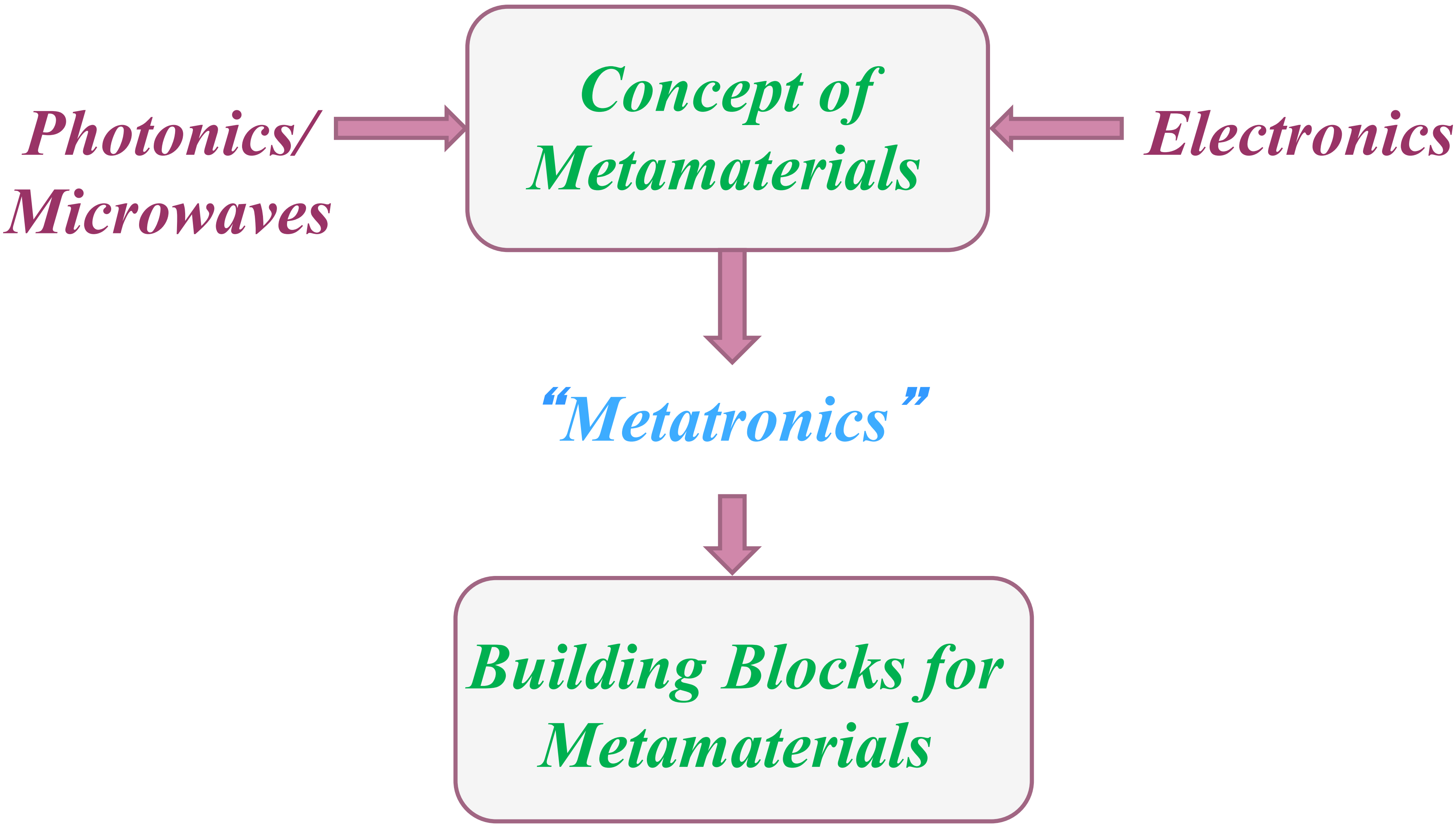
M

????

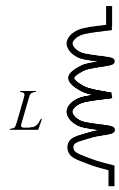
Metamaterial "Machines"?

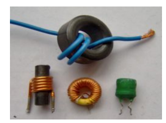


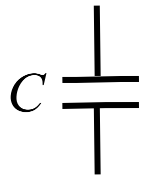
Cross Breeding: Photonics vs Electronics

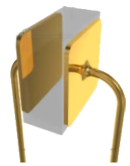


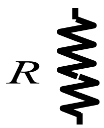
“Modular Blocks” in electronics

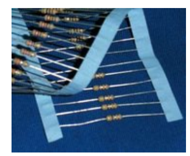
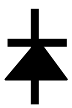


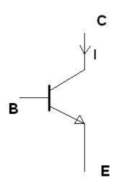


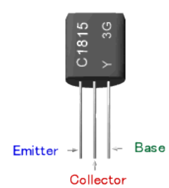




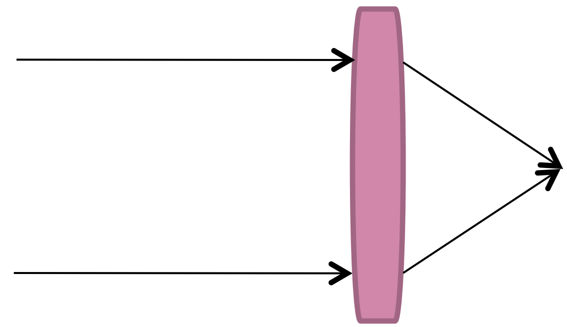








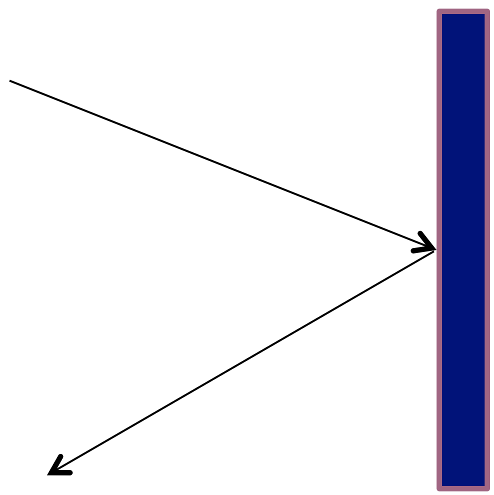
"Building Blocks" in Optics?

Waveguide

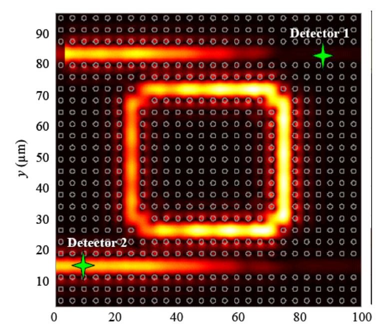


Lens

Optics

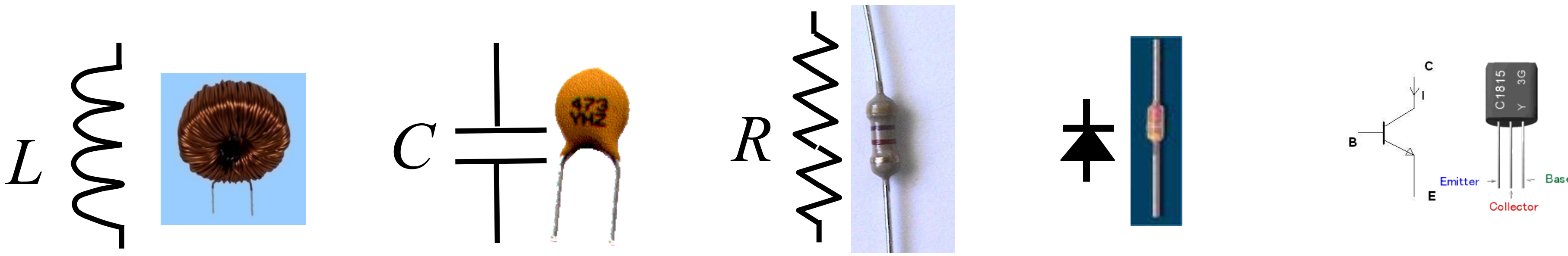


Mirror



from: D. Prather's group

"Lumped" Circuit Elements in Nanophotonics?



Radio Frequency (RF) electronics

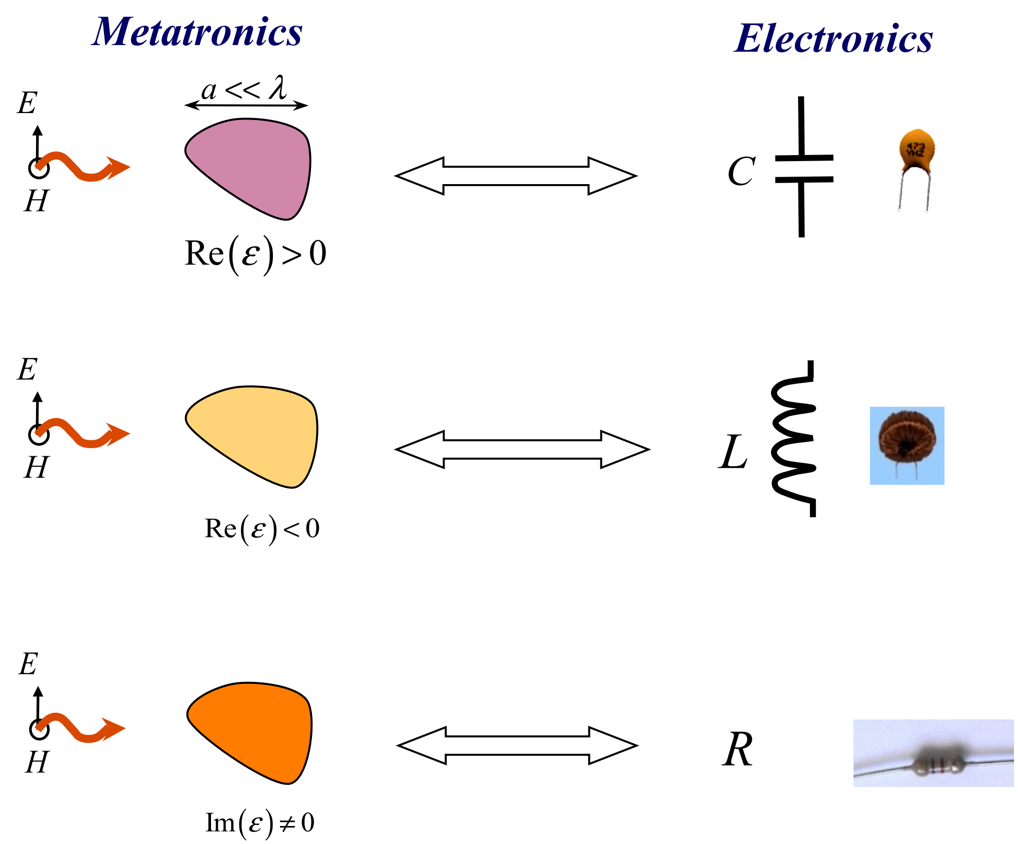
? ? ? ? ?

Nano-Optics

Optical Lumped Circuit Elements: Modular Blocks

$$\frac{\partial D}{\partial t} = -i\omega\varepsilon E$$

$$Z = \frac{Optical\ Voltage(E)}{Optical\ Displacement(D)}$$



Engheta, Science, 317, 1698 (2007)

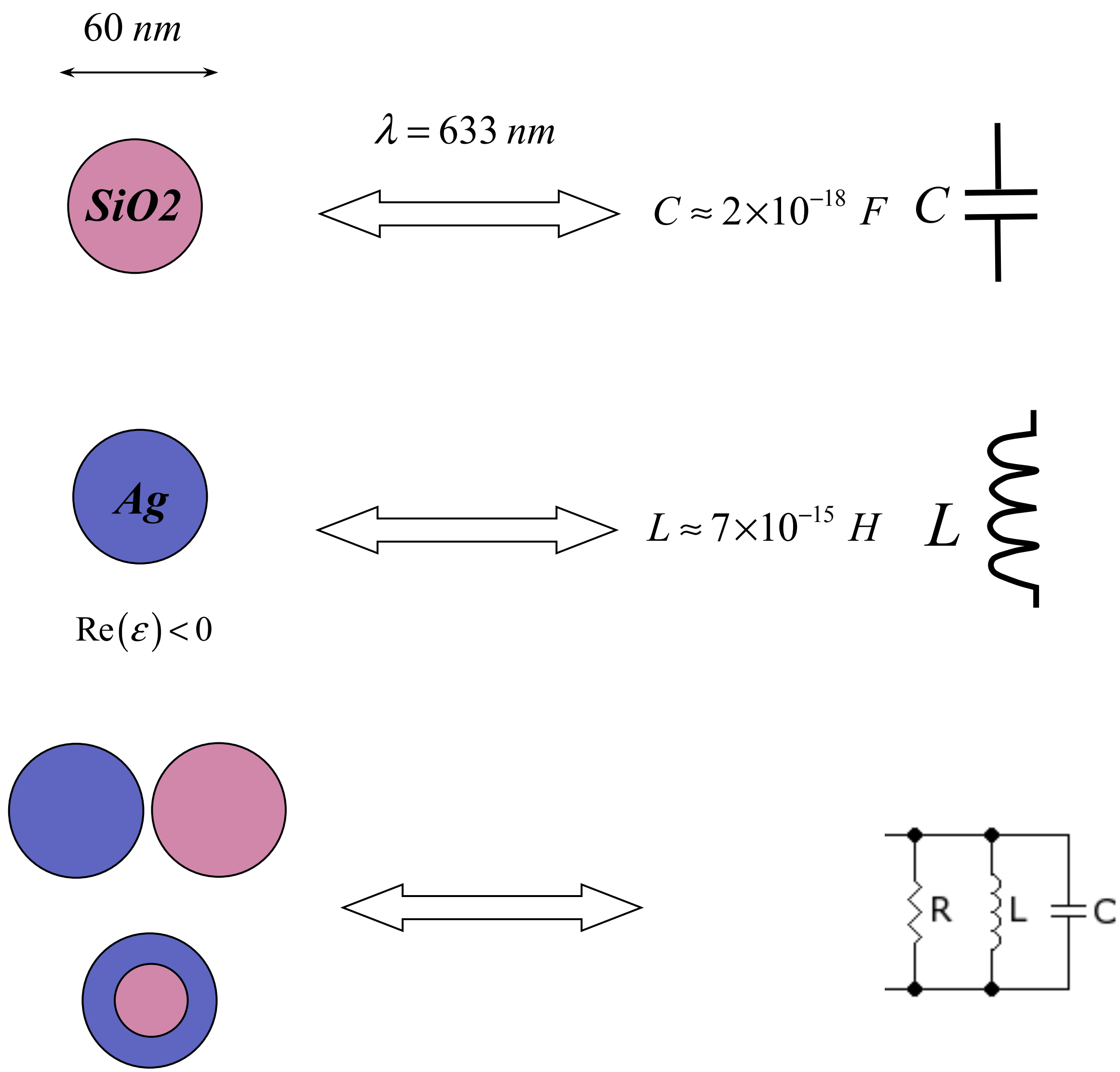
Engheta, Physics World, 23(9), 31 (2010)

Engheta, Salandrino, Alu, Phys. Rev. Lett. 95 (2005)

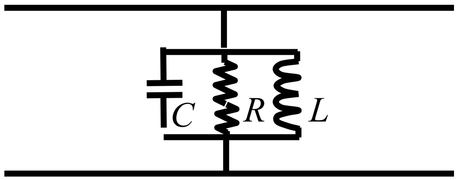
Caglayan, Hong, Edwards, Kagan, Engheta, Phys. Rev. Lett. (2013)

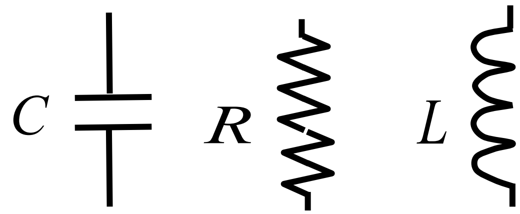
Sun, Edwards, Alu, Engheta, Nature Material, March 2012

Examples

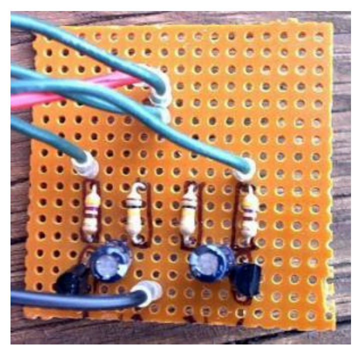


Electronic Circuit Design?

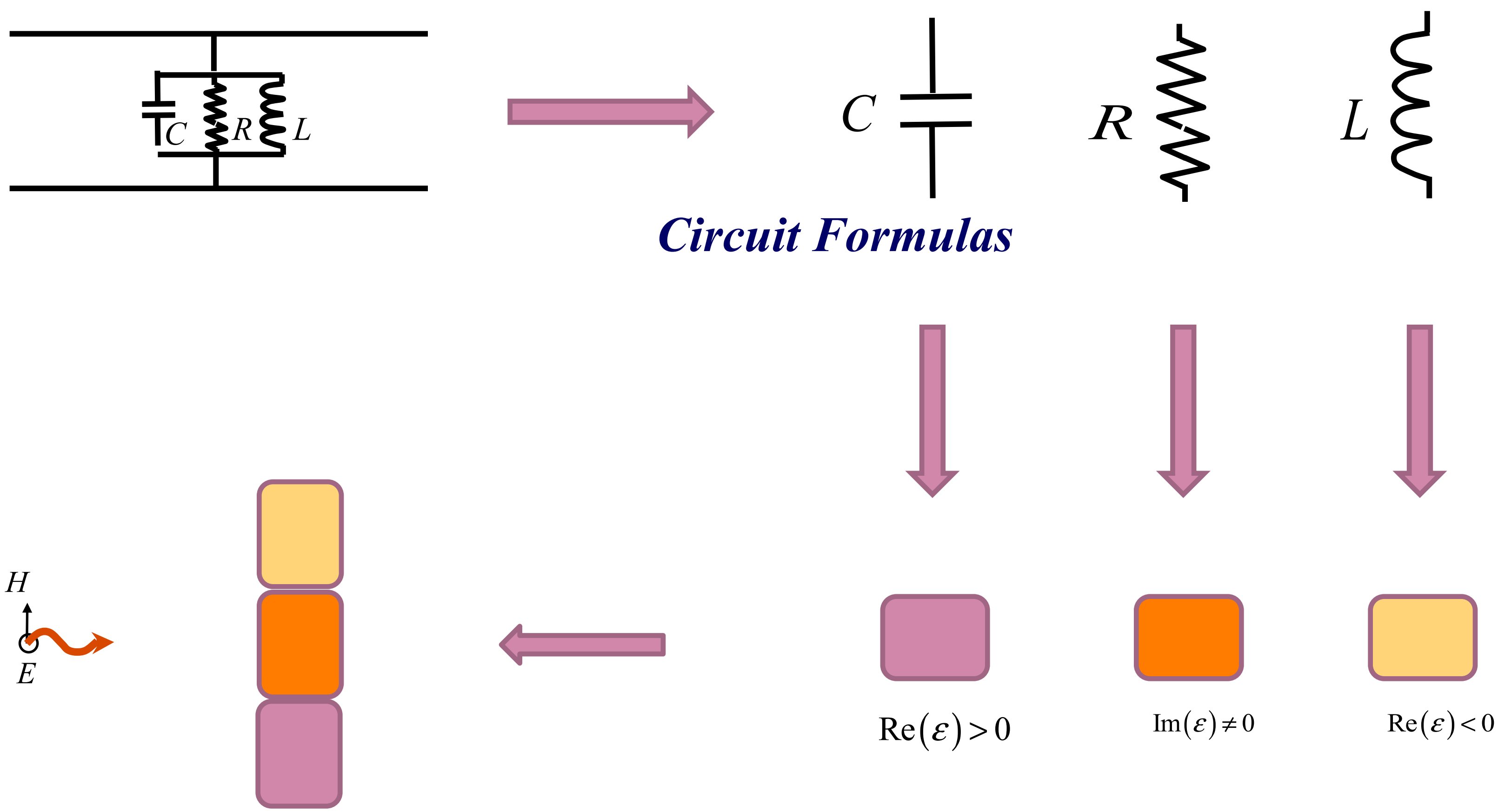




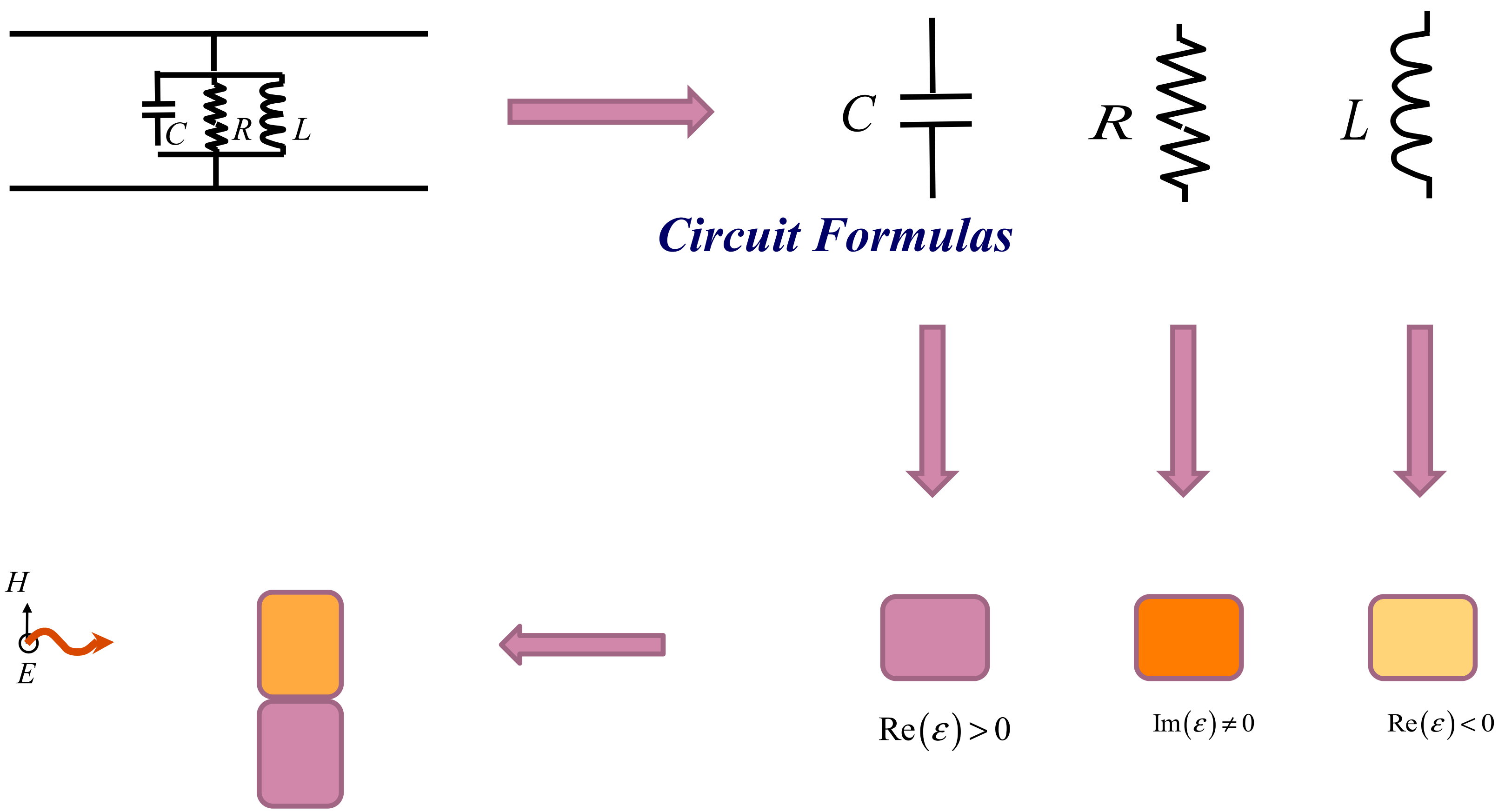
Circuit Formulas



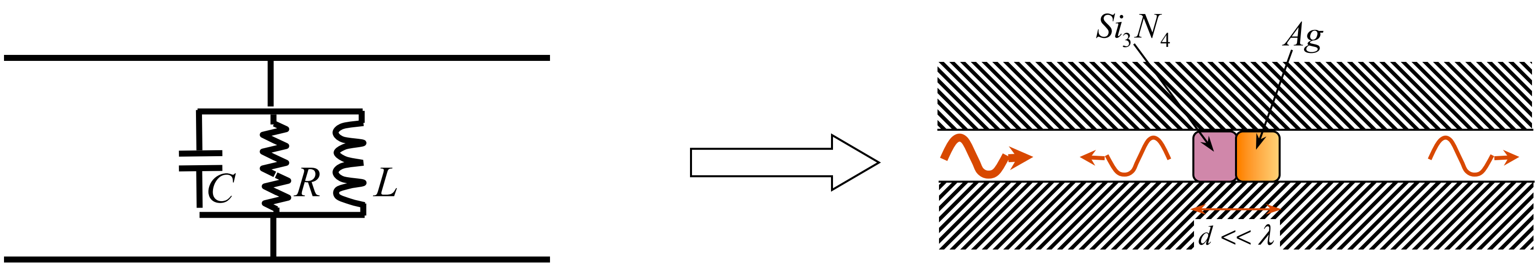
Can we do this in Nano-Optics?

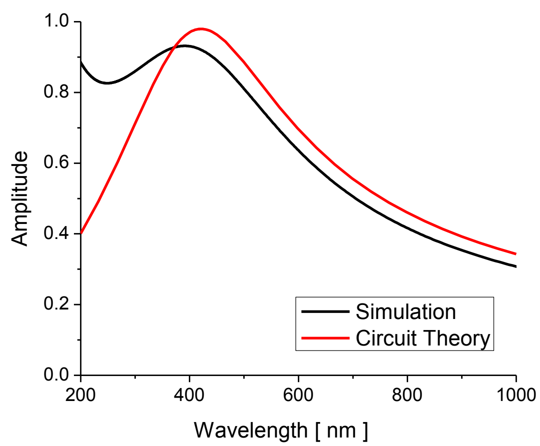


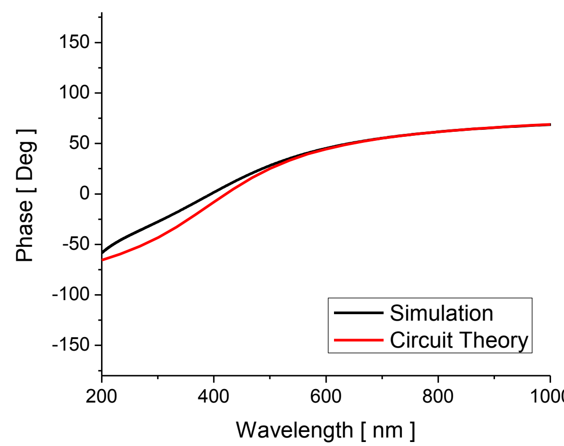
Can we do this in Nano-Optics?



Optical Filter with Nanorods



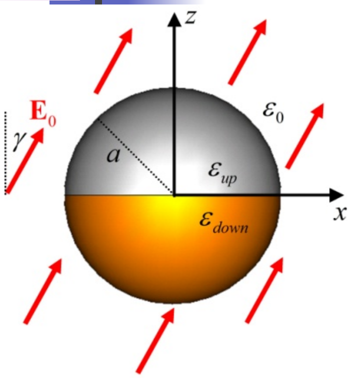


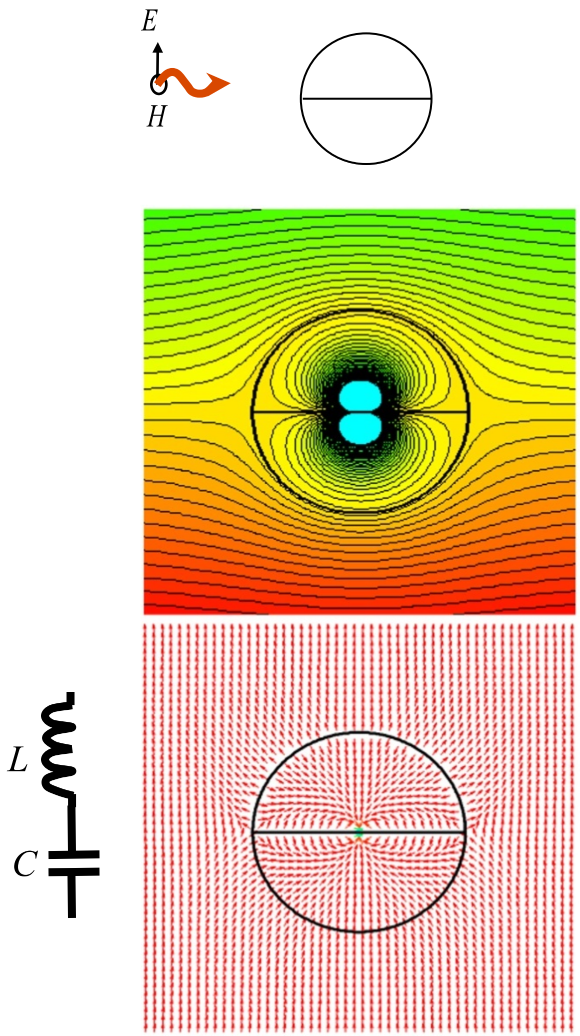


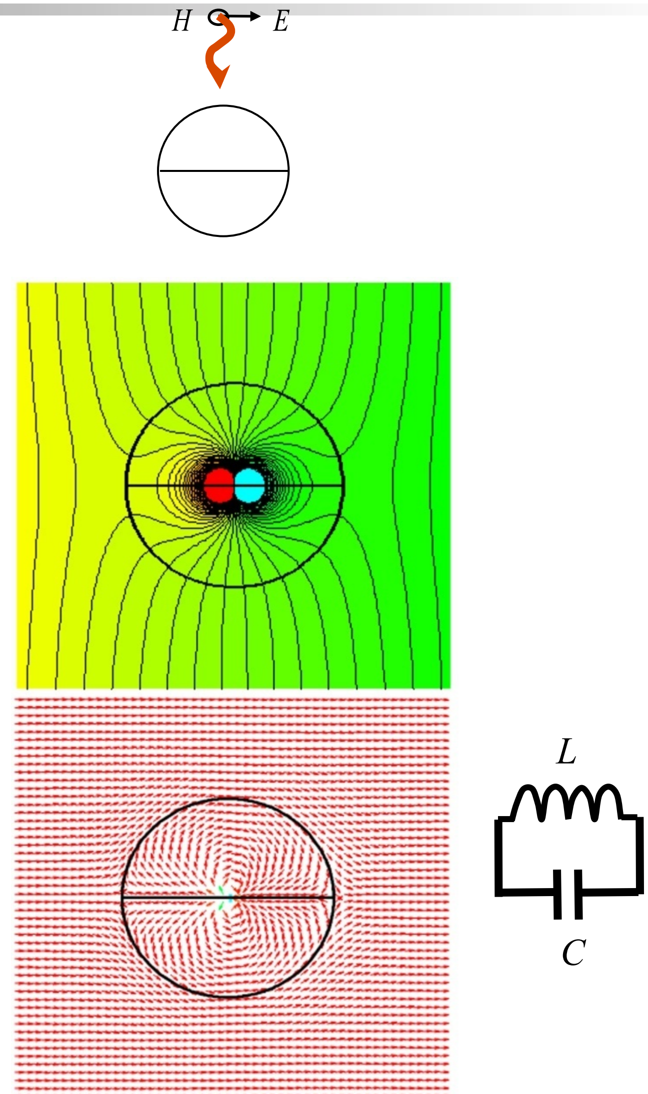
Engheta, Science, 317, 1698 (2007)

Alu, Young, and Engheta, Phys. Rev. B (2008)

"Stereo-Circuits"
Different "Circuits" for Different "Views"



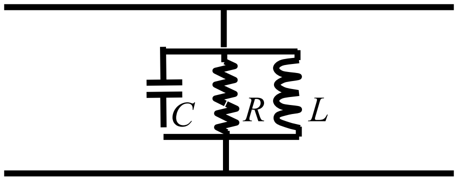


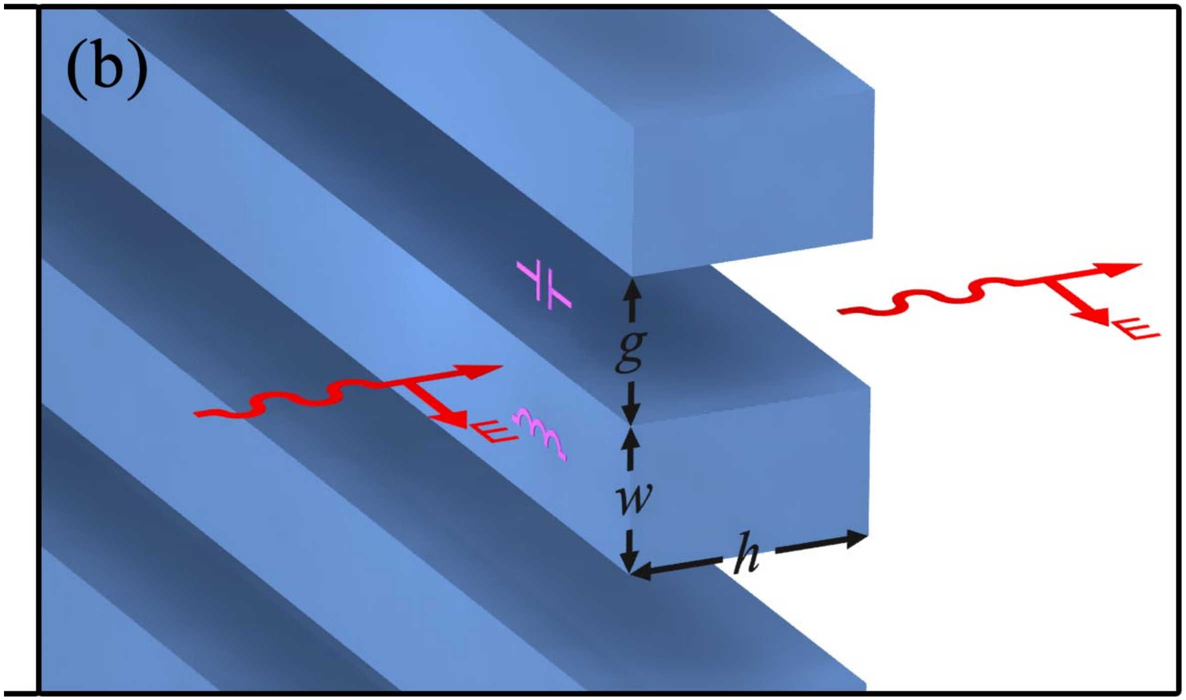


Salandrino, Alu, Engheta, JOSA B, Part 1, 2007
Alu, Salandrino, Engheta, JOSA B, Part 2, 2007

Alu and Engheta, New Journal of Physics, 2009

Experimental Verification at IR





$W = 75nm, 125nm, 225nm$

$g = 75nm$

$h = 175nm, 250nm, 325nm$

Y. Sun, B. Edwards, A. Alu, and N. Engheta, Nature Materials, March 2012

Experimental Verification at IR

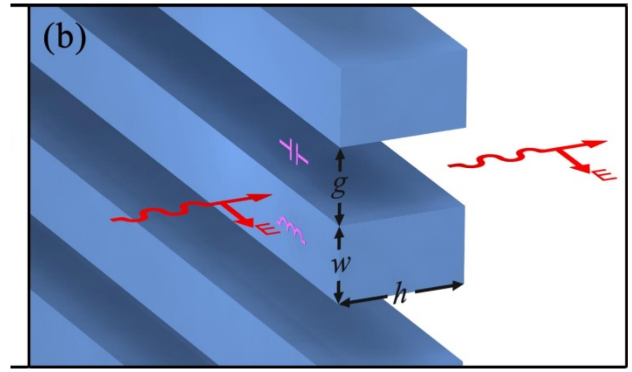
Circuit Theory Model

$$Z_{wire}^{par} \equiv \frac{i}{\omega h w \varepsilon_{Si_3N_4}}$$

$$Z_{air\text{-}gap}^{par} \equiv \frac{i}{\omega h g \varepsilon_{air}}$$

$$Z_{equivalent}^{par} \equiv \frac{Z_{wire}^{par} \cdot Z_{air\text{-}gap}^{par}}{Z_{wire}^{par} + Z_{air\text{-}gap}^{par}}$$

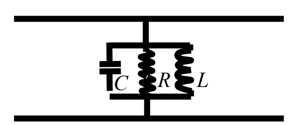
$$T^{par} = \left| \frac{Z_{equivalent}^{par}}{Z_{equivalent}^{par} + \left[\eta_o / \left(2 \left(W + g \right) \right) \right]} \right|^2$$



$$g = 75nm$$

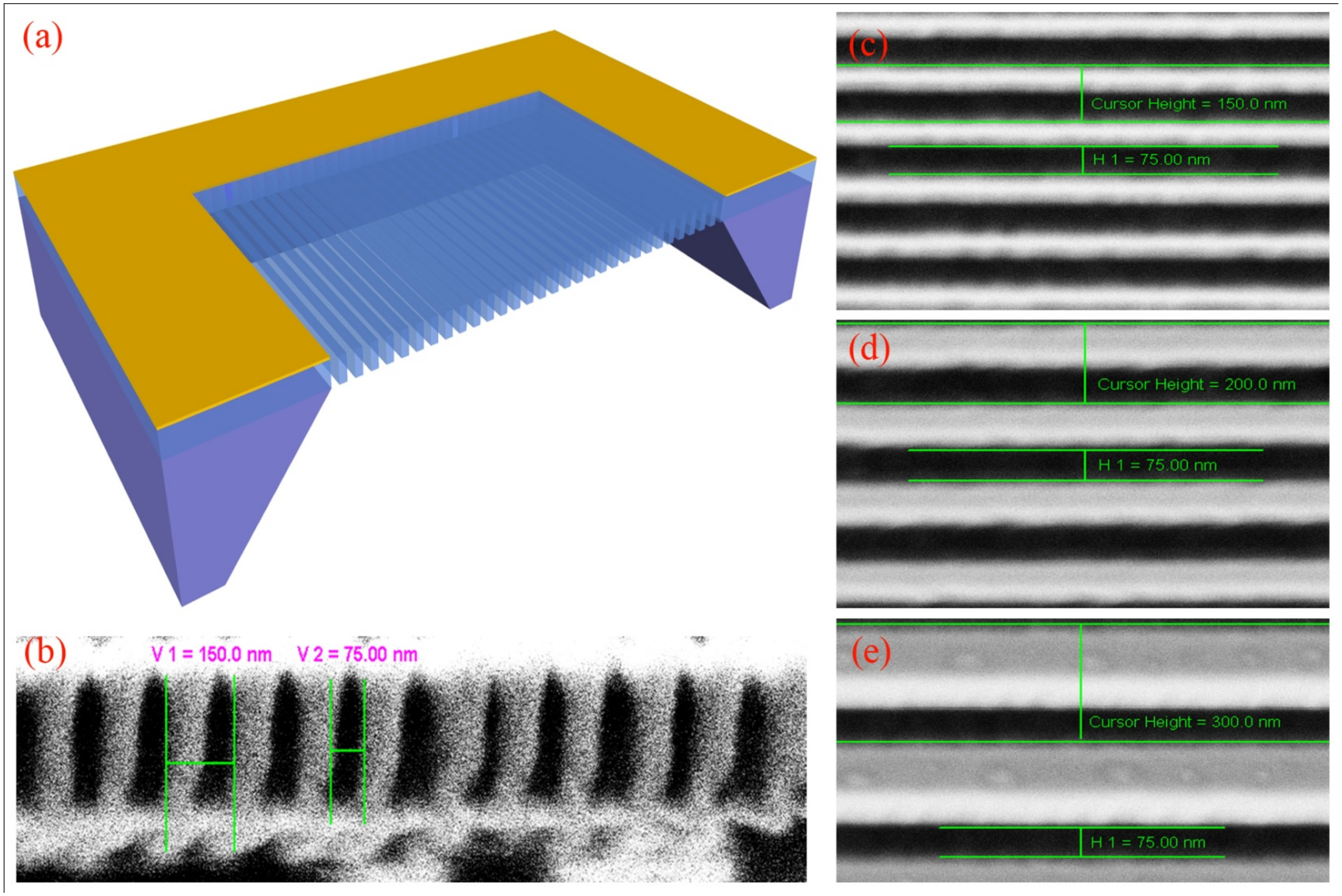
$$h = 175nm, 250nm, 325nm$$

$$W = 75nm, 125nm, 225nm$$



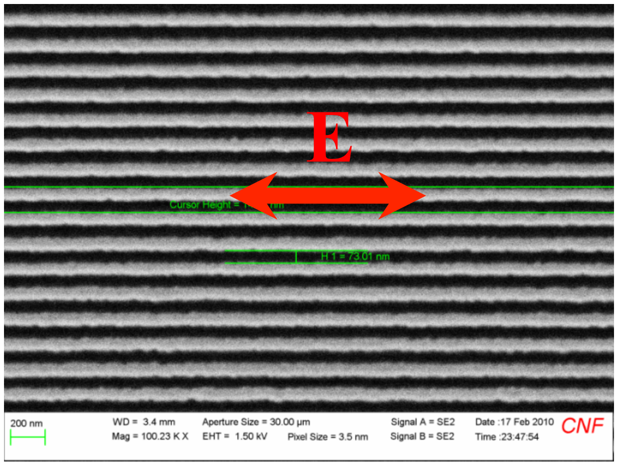
Y. Sun, B. Edwards, A. Alu, and N. Engheta, Nature Materials, March 2012

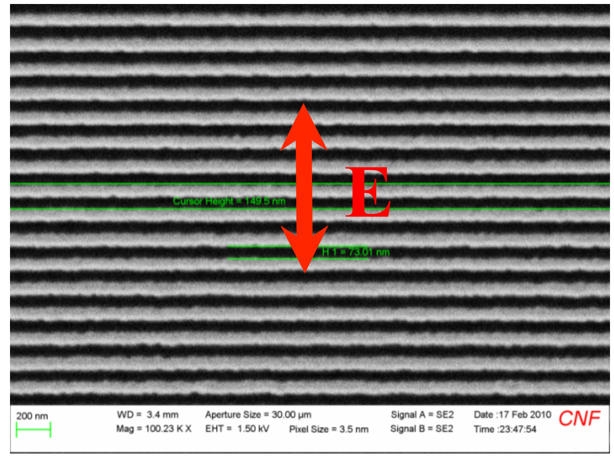
Our Samples

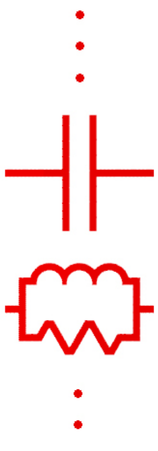


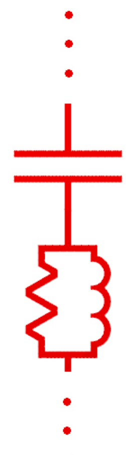
Y. Sun, B. Edwards, A. Alu, and N. Engheta, Nature Materials, March 2012

“Parallel” and “Series” Optical Circuits



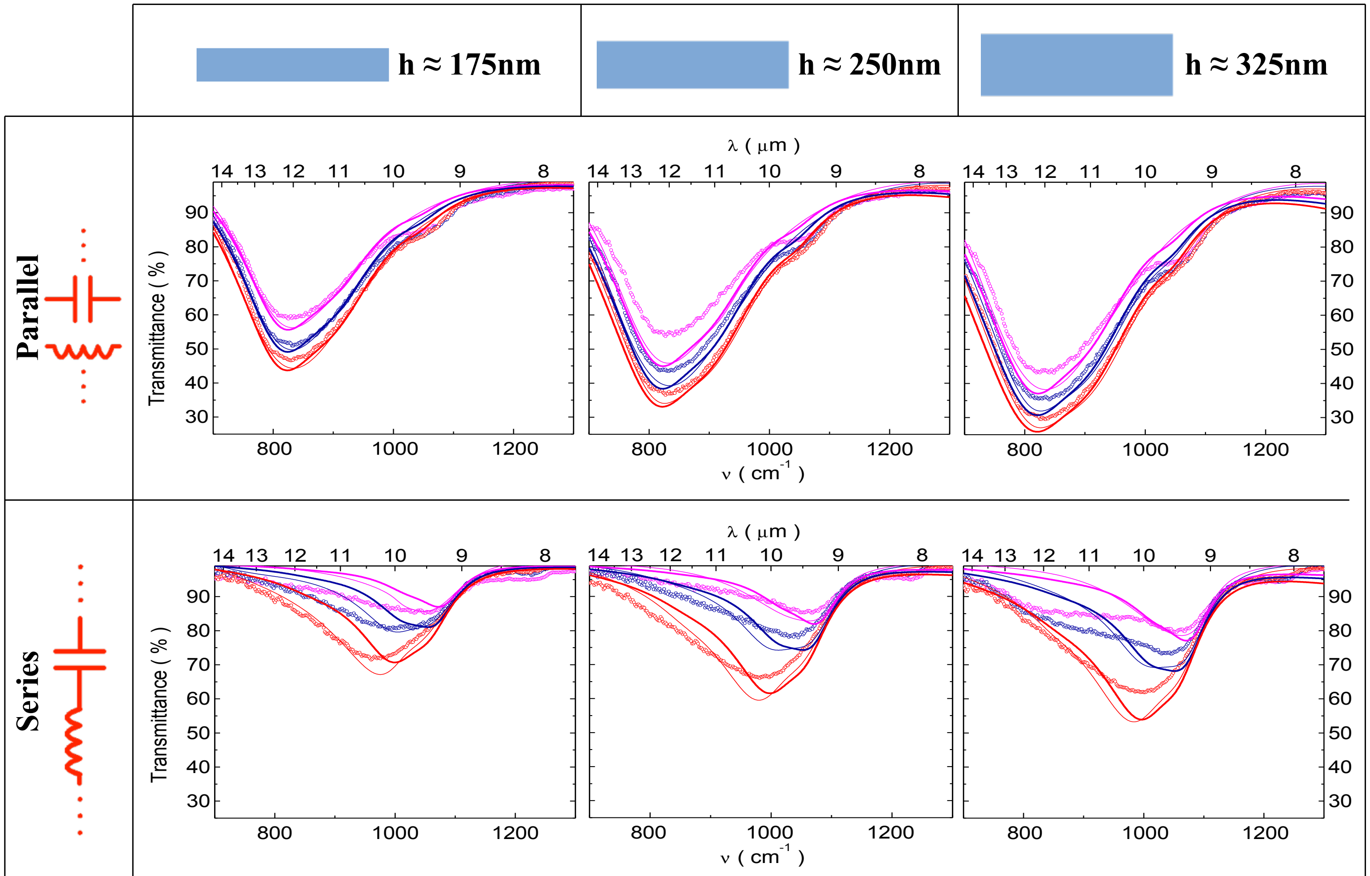






Y. Sun, B. Edwards, A. Alu, and N. Engheta, Nature Materials, March 2012

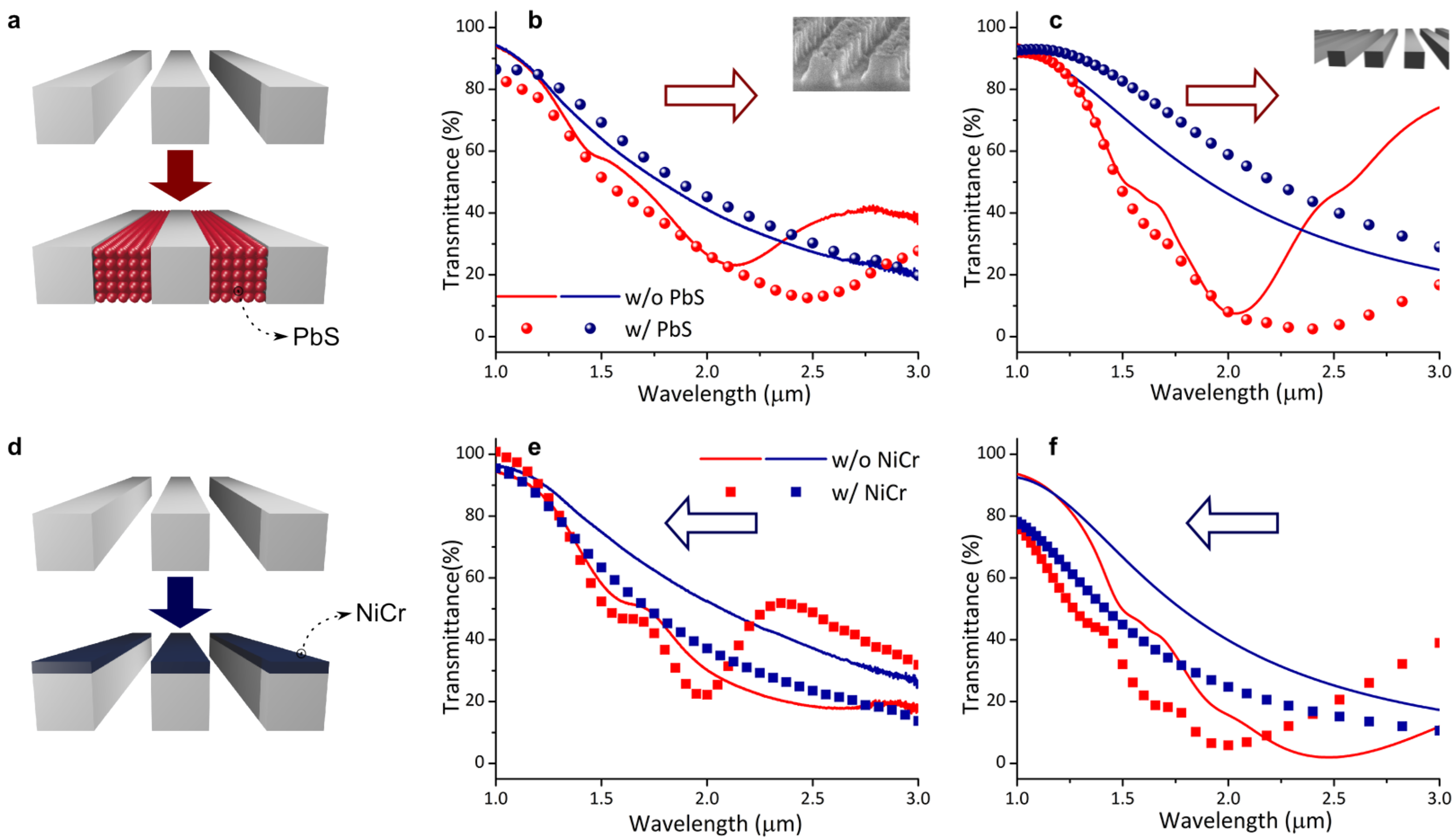
Collective Results



"empty circle": Experimental data	Magenta: w ~ 75 nm, g ~ 75 nm
"Thick line": Circuit theory	Royal blue: w ~ 125 nm, g ~ 75 nm
"Thin line" : Full wave simulation	Red: w ~ 225 nm, g ~ 75 nm

Y. Sun, B. Edwards, A. Alu, and N. Engheta, Nature Materials, March 2012

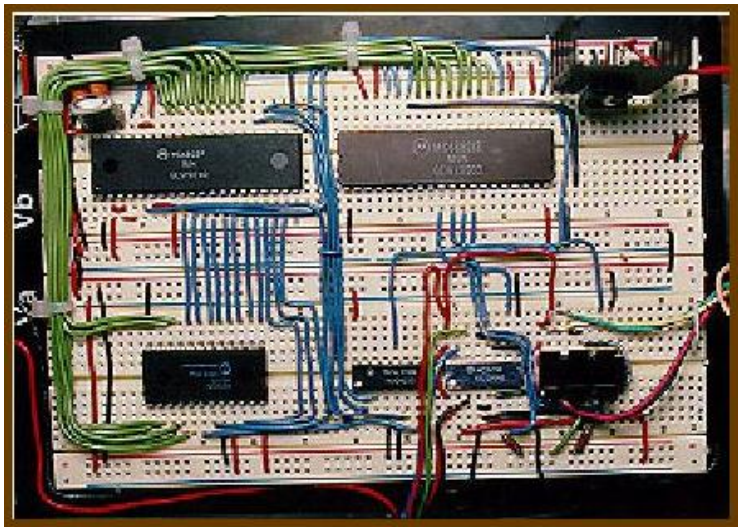
TCO NIR Metatronic Circuits
Fabrication and Experimental Results



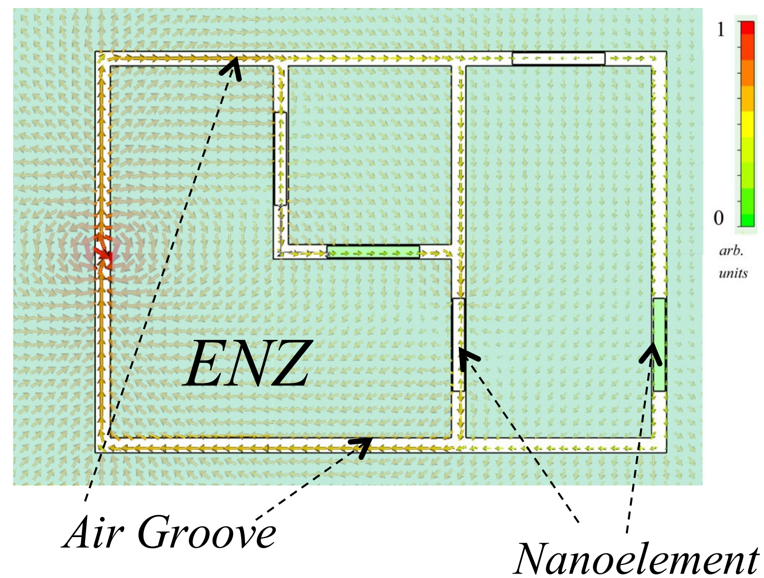
Caglayan, Hong, Edwards, Kagan, Engheta, Phys. Rev. Lett. 111, 073904 (2013)

Nano-Optics Circuit Boards

Electronic Circuit Board

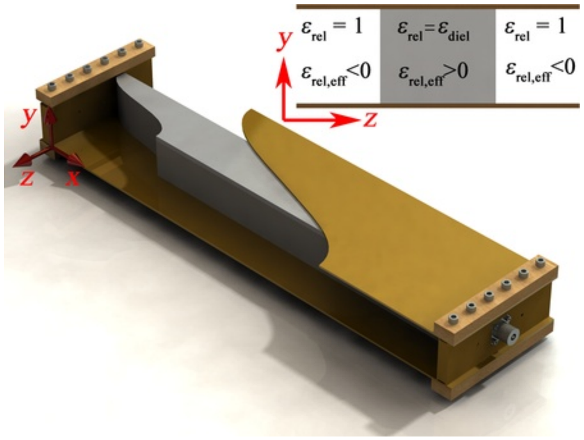


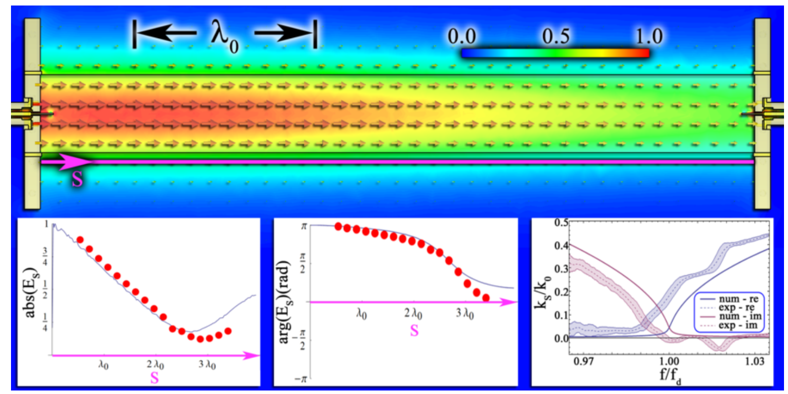
Metatronic Circuit Board

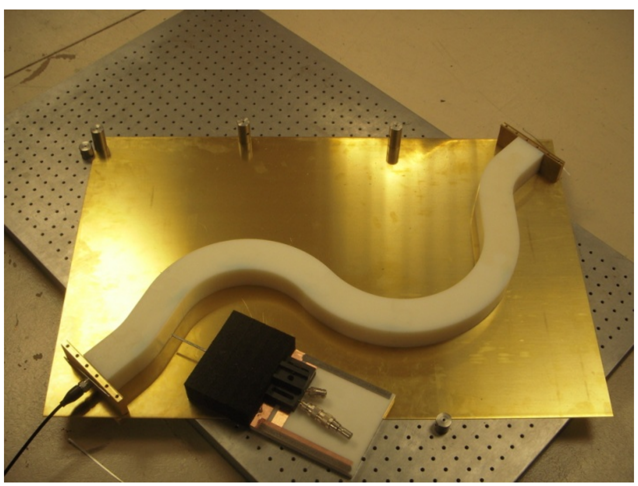


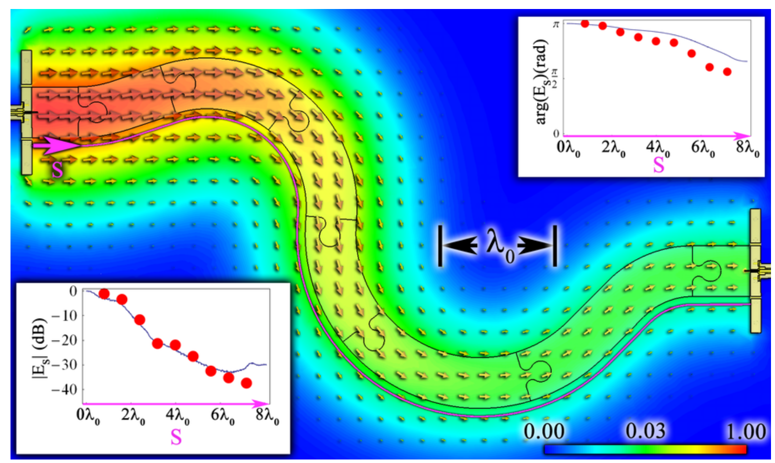
Alu and Engheta, Phys. Rev. Lett., 2009

Experimental Verification of Displacement-Current Wire



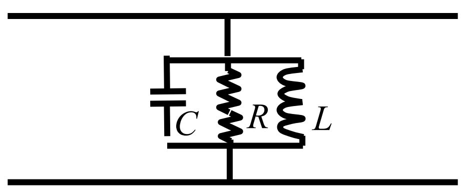


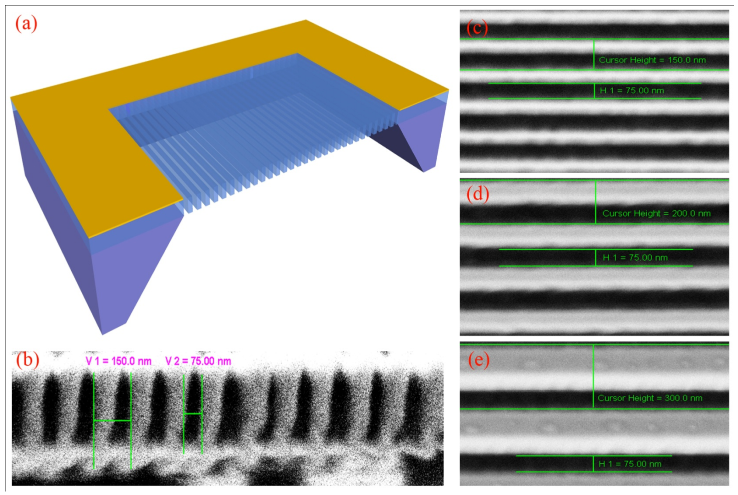




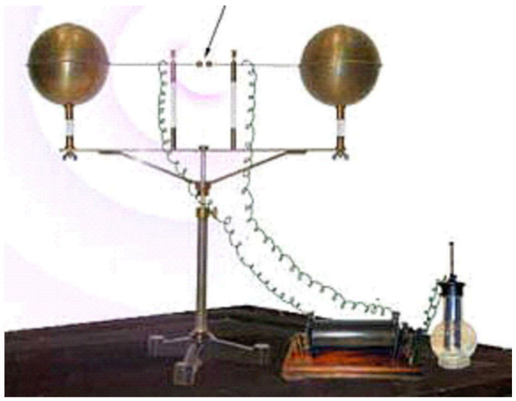
B. Edwards and N. Engheta, Physical Review Letters, May 7, 2012

From a "Filter" to a "Filter"

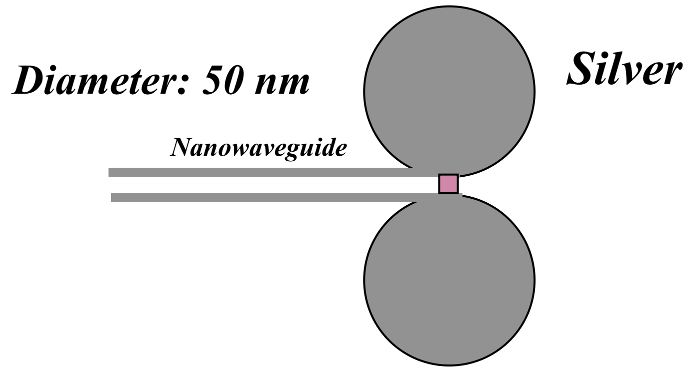


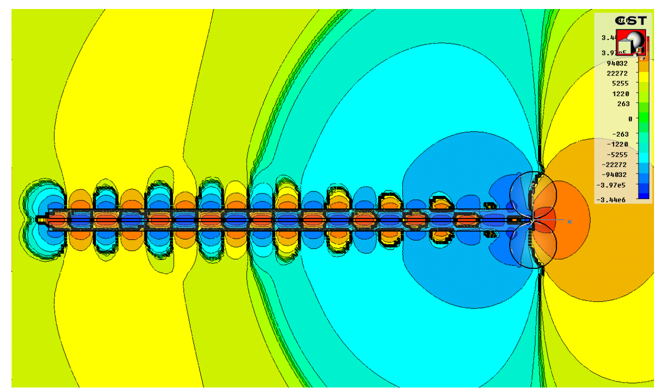


From an "Antenna" to an "Nanoantenna"

a) 

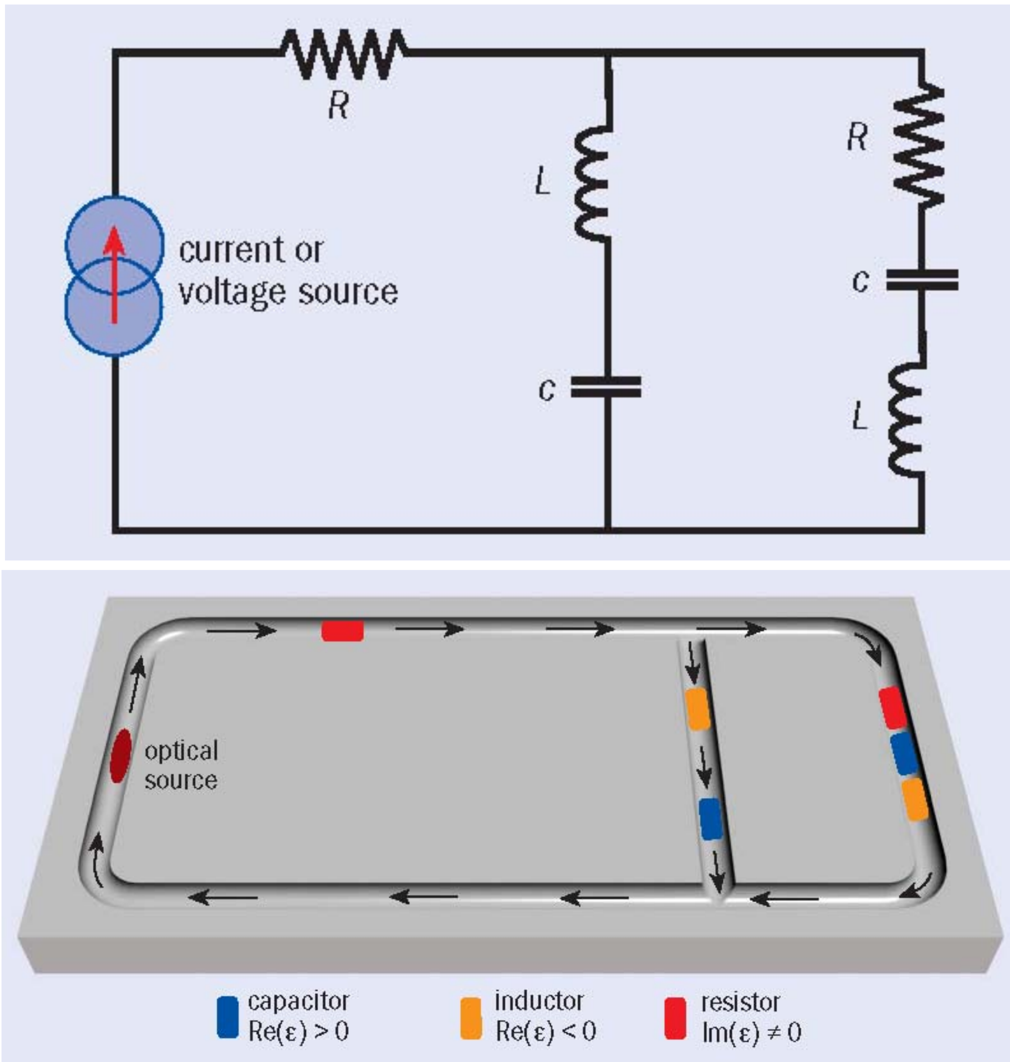
From: http://www.sparkmuseum.com



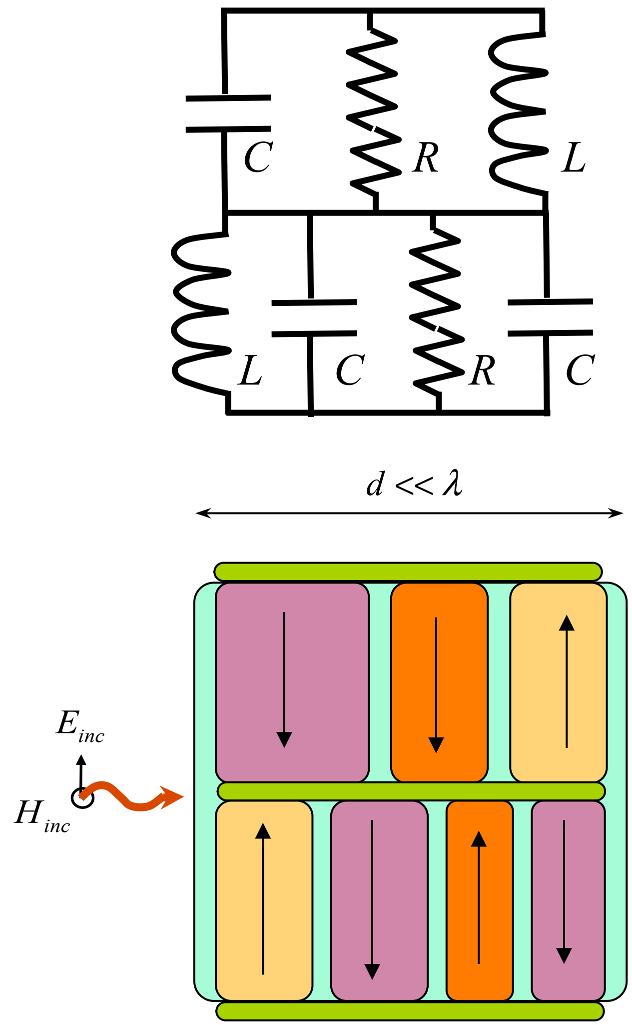


A. Alu and N. Engheta, Phys. Rev. B. 2008

Optical Metatronics



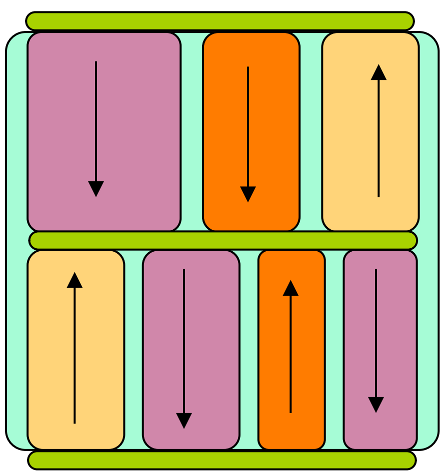
Engheta, Physics Worlds, 23(9), 31 (2010)

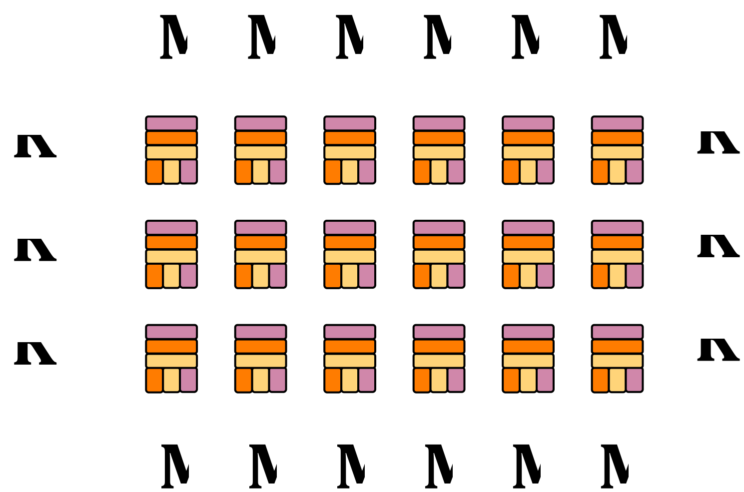


Engheta, Science, 317, 1698 (2007)

Metatronics vs Metamaterials

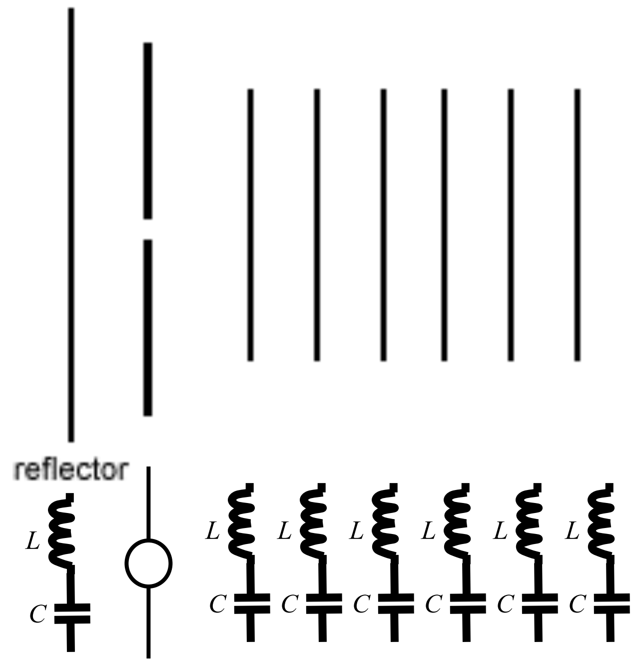
Metatronics ⟷ *Building Blocks for Metamaterials*





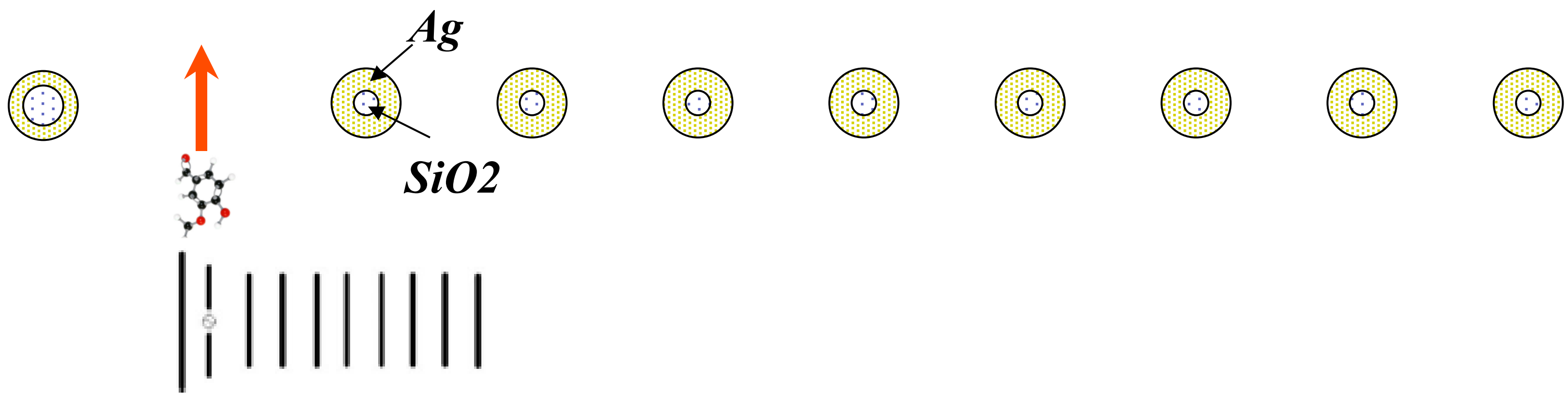
Yagi-Uda Antennas

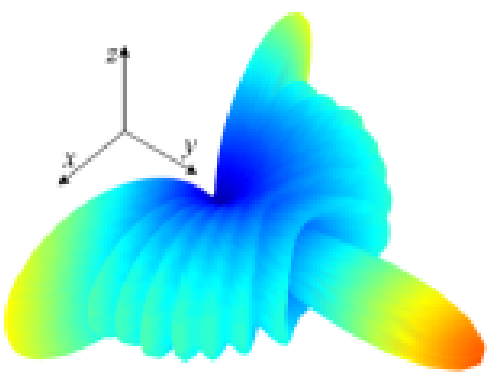
Picasaweb.google.com/.../YOKis5Vf7nhDG5dGAoSD0w



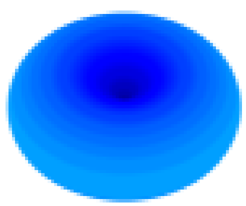
Li, Salandrino, and Engheta, Phys. Rev. B , 76, 245403 (2007)

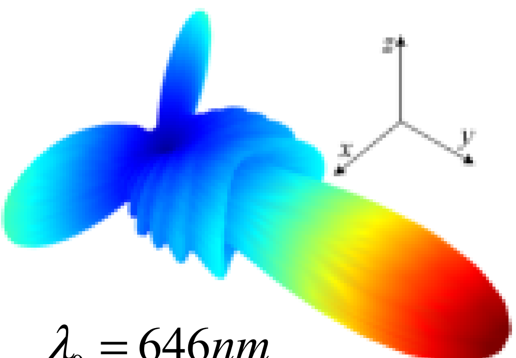
Optical "Yagi-Uda" Nanoantenna





$\lambda_0 = 620nm$



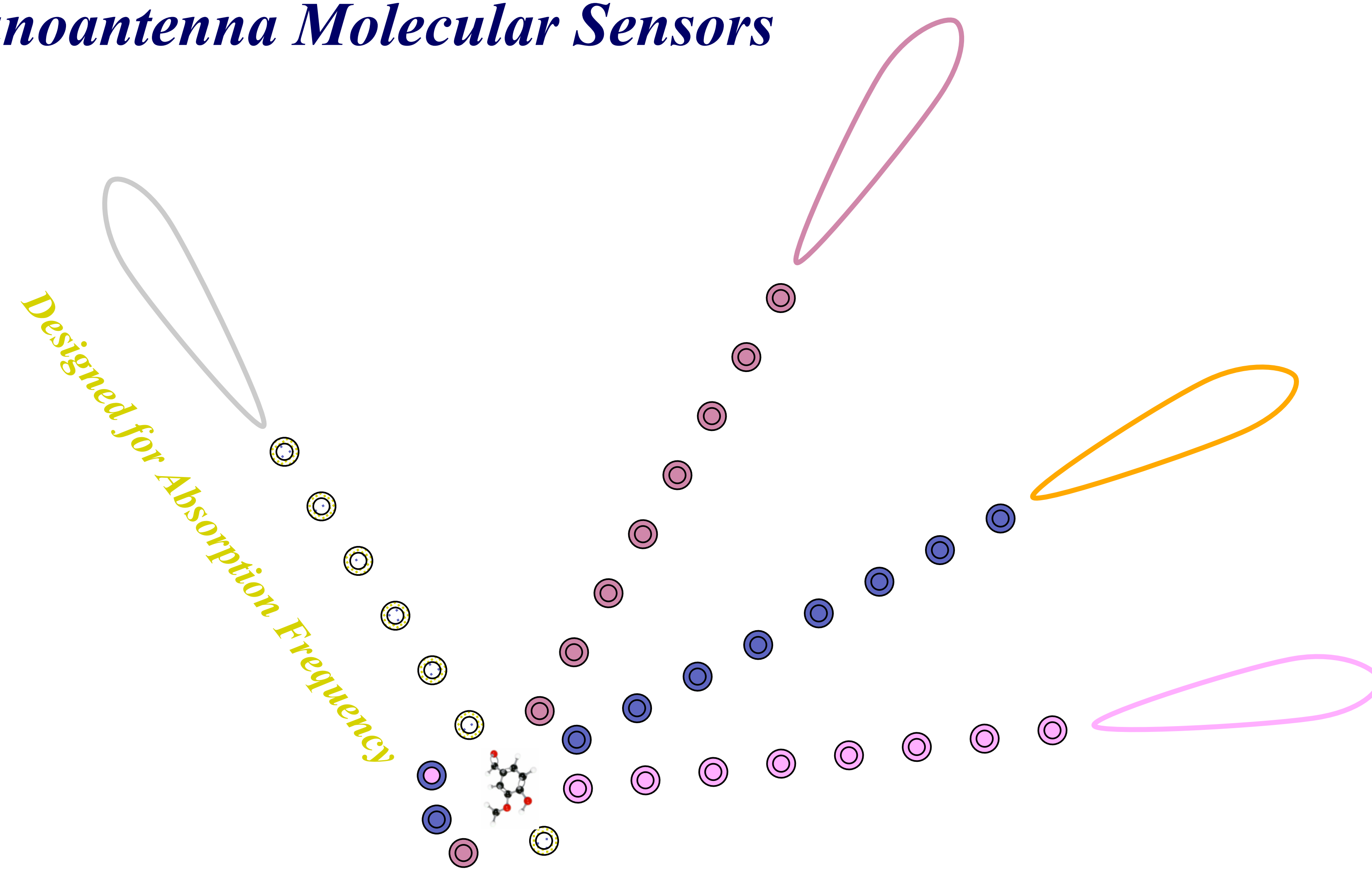


$\lambda_0 = 646nm$

Li, Salandrino, and Engheta, Phys. Rev. B , 76, 245403 (2007)

Nanoscale "Spectrometer" in Molecular Spectroscopy

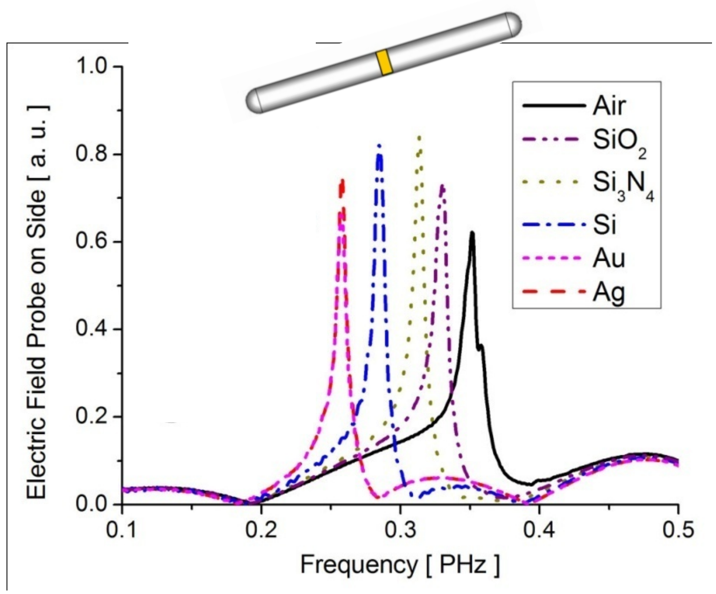
Nanoantenna Molecular Sensors



Li, Salandrino, and Engheta, Phys. Rev. B, 2009

Optical Wireless Link at Nanoscales

Antennas, local oscillators, filters, switches, mixers, modulators, demodulators, etc. etc.

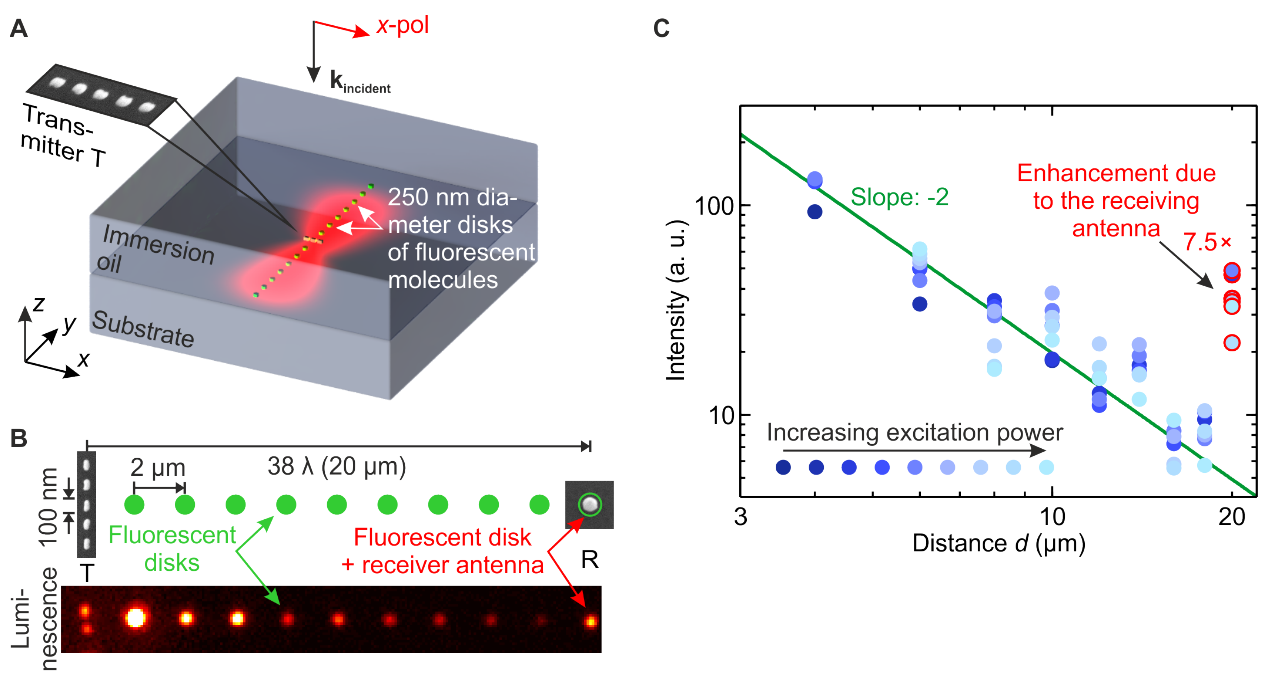


Alu and Engheta, Phys. Rev. Lett, May 2010

Alu and Engheta, Nature Photonics, Vol. 2, May 2008

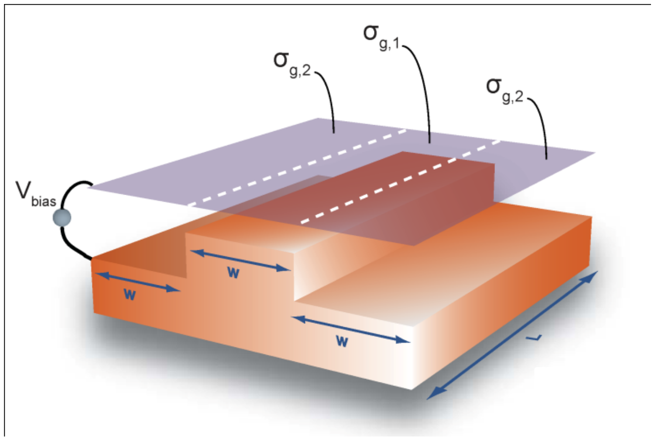
Experimental Verification

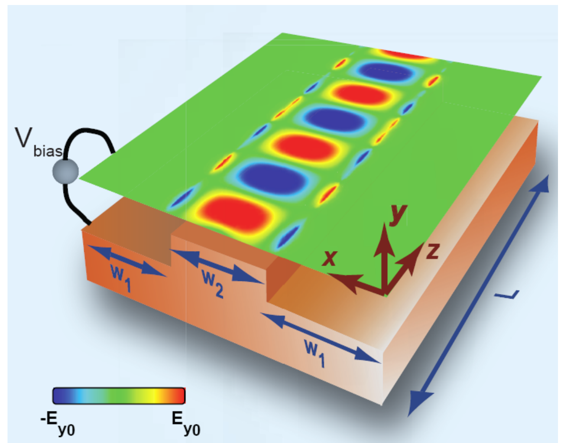
Harald Giessen's group in collaboration with my group

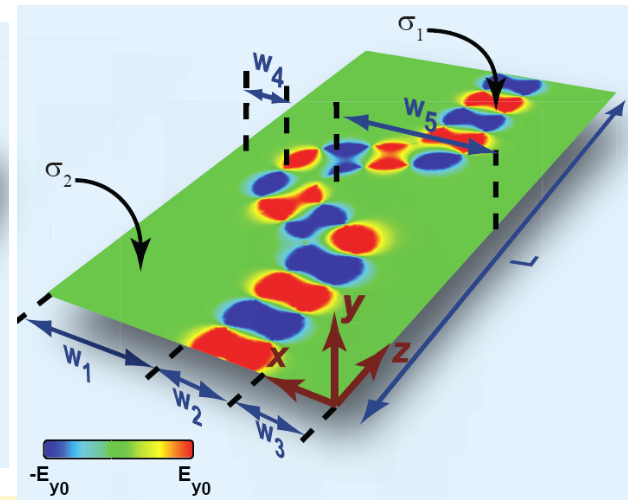


D. Dregely, K. Lindfors, M. Lippitz, N. Engheta, M. Totzeck, H. Giessen, <u>Nature Communications, 2014</u>

One-Atom-Thick Optical Devices





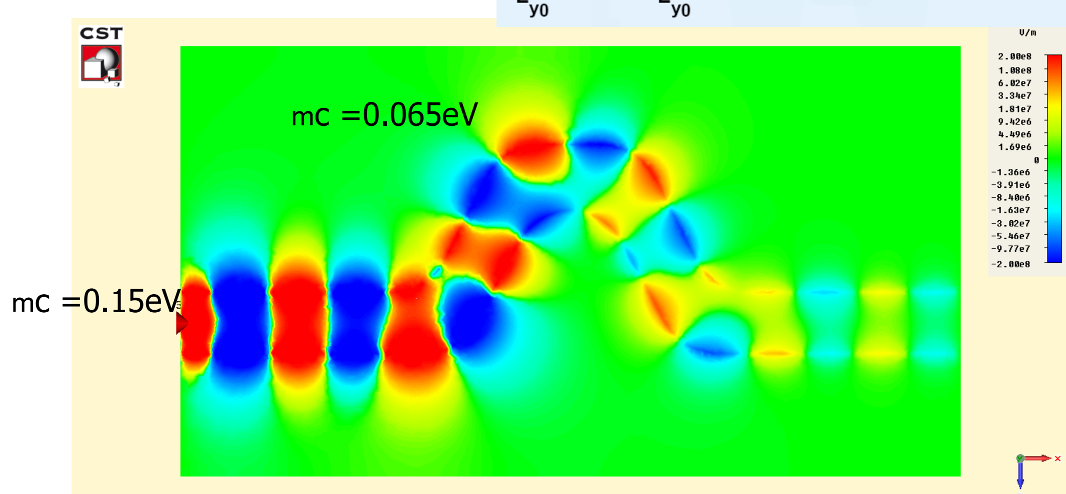


Region 1: $\sigma_{g,i} > 0$

$\mu_c = 150\, meV$

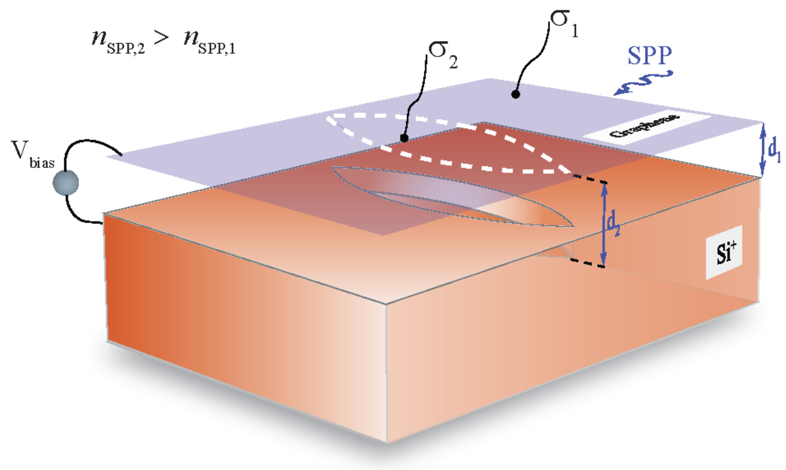
Region 2: $\sigma_{g,i} < 0$

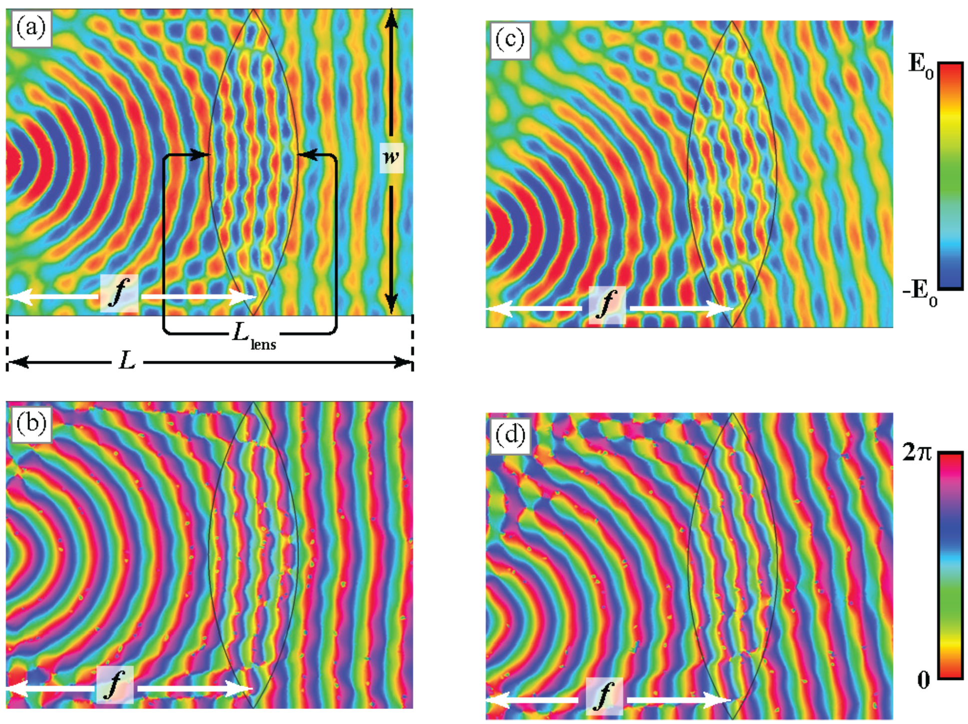
$\mu_c = 65\, meV$



A. Vakil and N. Engheta, Science, 2011

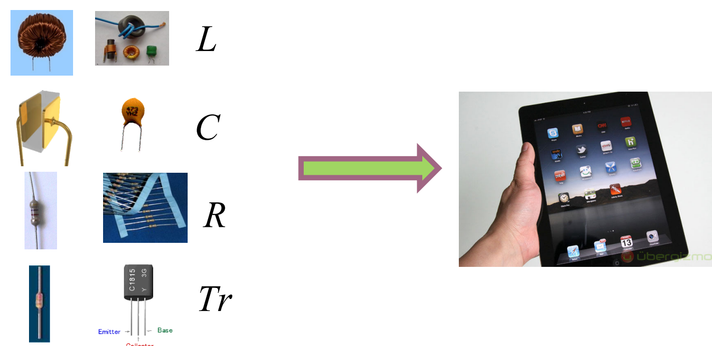
One-Atom-Thick Signal Processing: Fourier Transform





Vakil, Engheta, Phys. Rev. B, (2012)

Metasystems



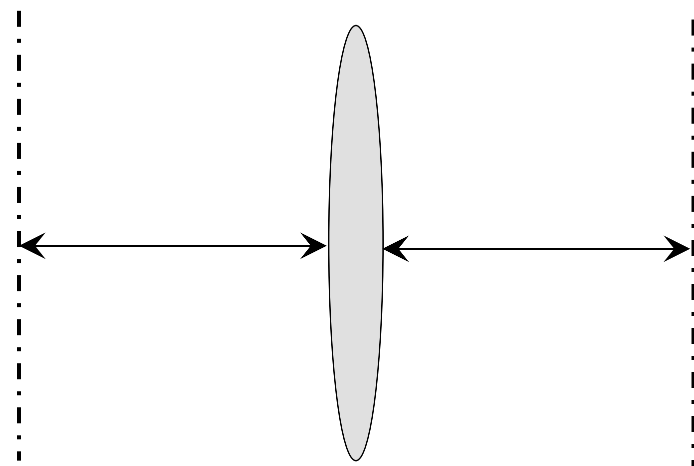
Signal-Processing Metamaterials?

Metamaterial Computing

$$g(x_1, x_2, \cdots) = \iiint f(u_1, u_2, \cdots) k(x_1, x_2, \cdots; u_1, u_2, \cdots) du_1 du_2 \cdots$$

Metamaterial Analog Computer?

Fourier-Transform



$f(x, y)$ $\qquad$ $F(\bar{x}, \bar{y})$

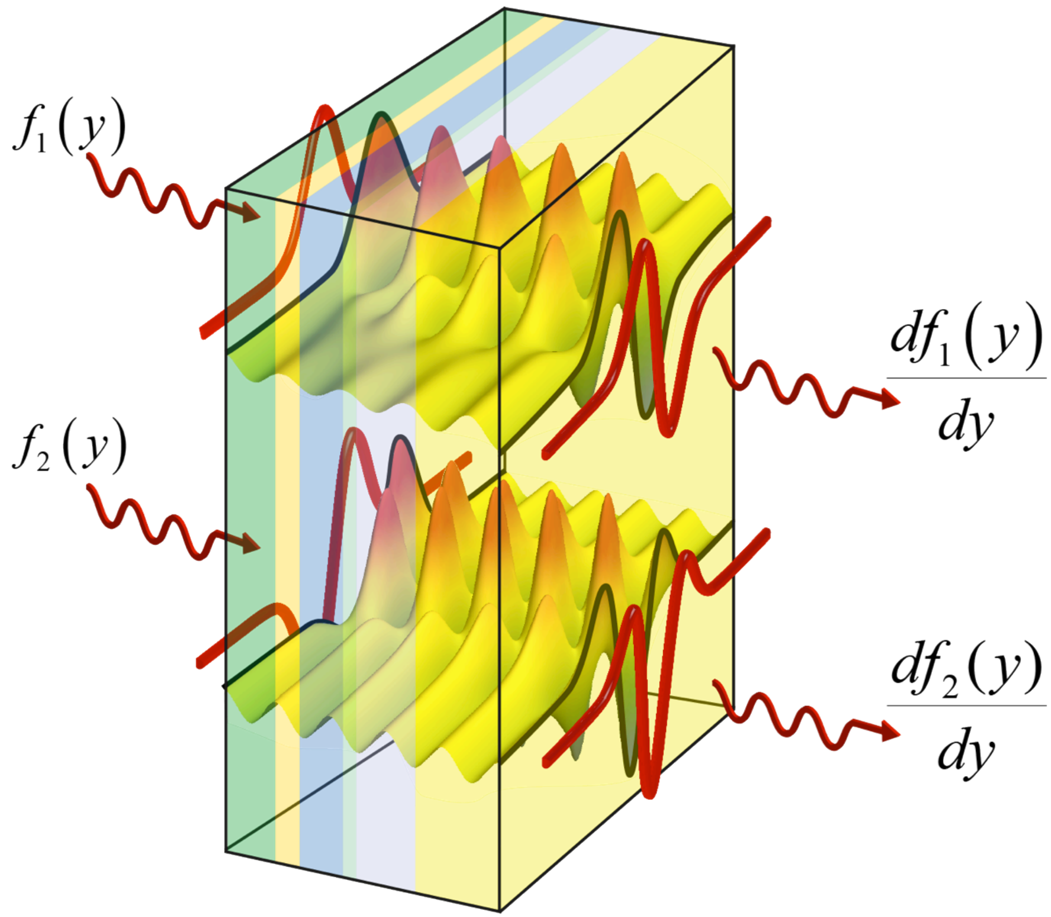
$$F(\bar{x}, \bar{y}) : \ Fourier\ Transform\left[f(x, y)\right]$$

J. Goodman, Fourier Optics, 1994

Metamaterial Computing

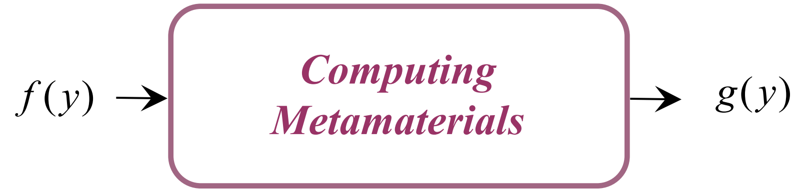
"Differentiator" Metamaterial

Computing Metamaterial



"Differentiator" Metamaterial

$$g(y) \sim \frac{df(y)}{dy}$$

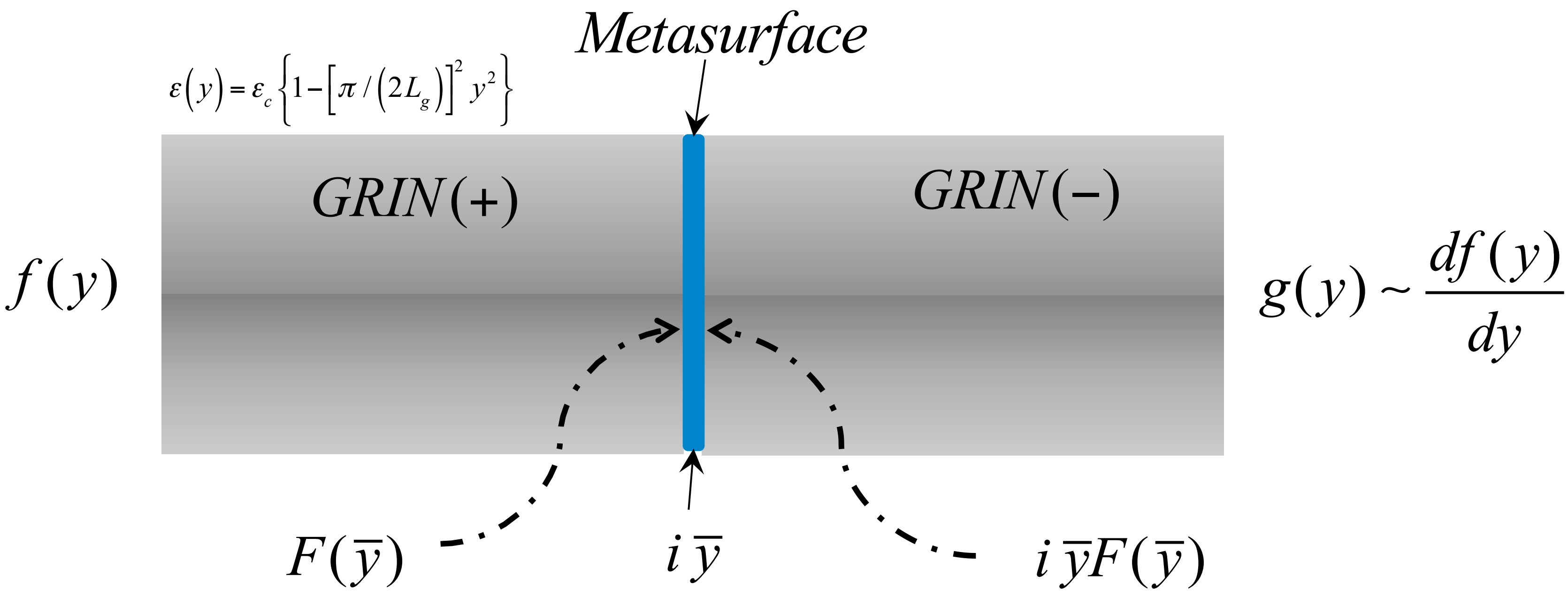


$$f(y) \xrightarrow{Fourier} F(\overline{y})$$

$$g(y) \xrightarrow{Fourier} G(\overline{y})$$

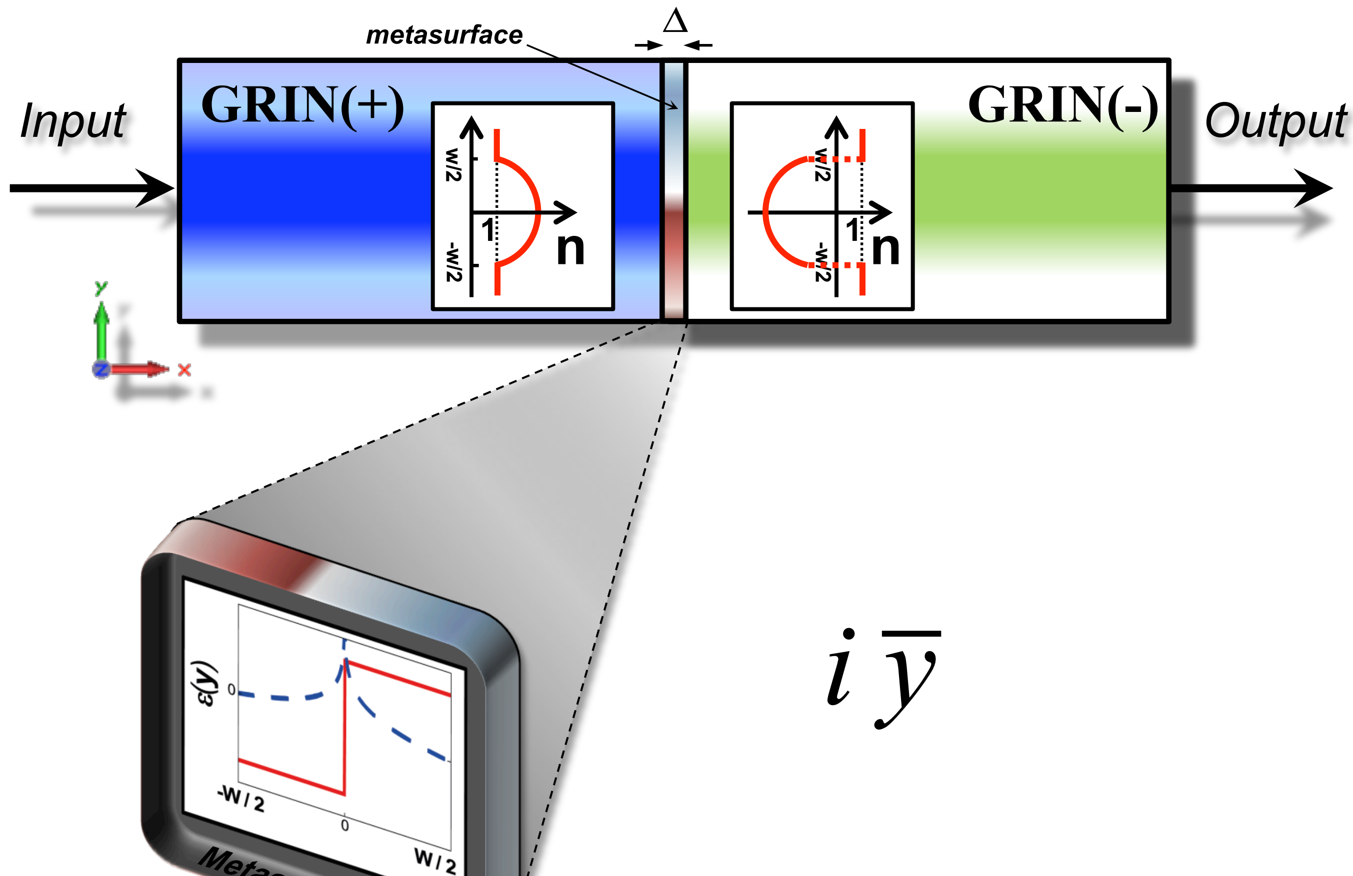
$$G(\overline{y}) \propto \left(i\overline{y}\right) F(\overline{y})$$

"Differentiator" Metamaterial



A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

GRIN(+) - MS - GRIN (-)

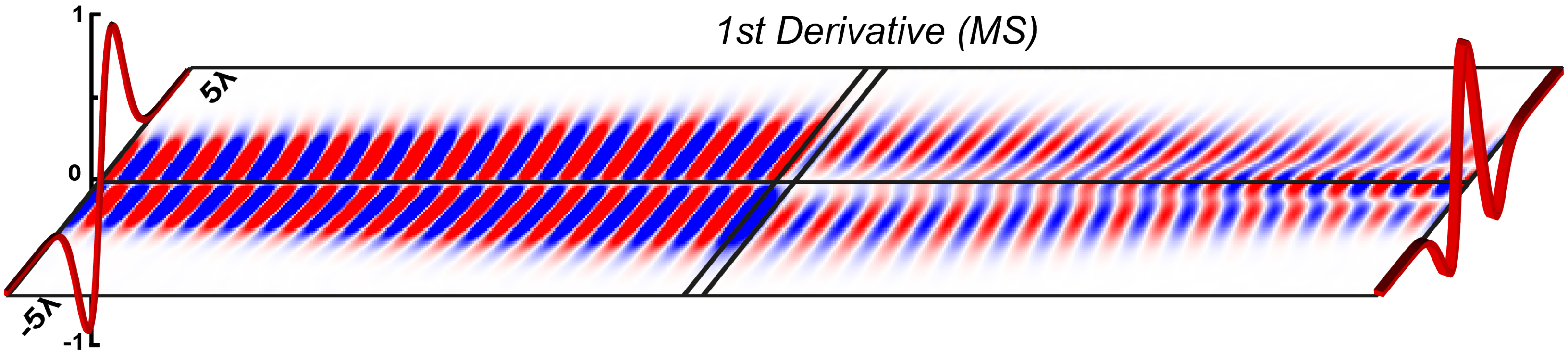


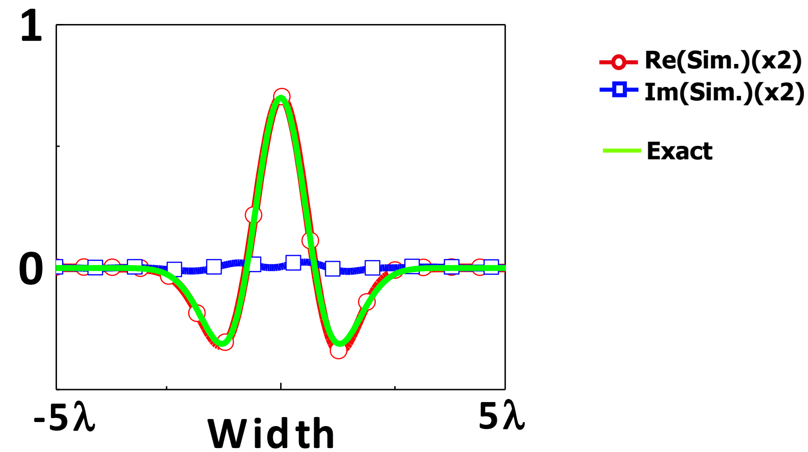
$i\,\bar{y}$

$$\varepsilon_{ms}(y)/\varepsilon_o = \mu_{ms}(y)/\mu_o = i\left[\lambda_o/(2\pi\Delta)\right]\ln\left(-iW/(2y)\right)$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

Metamaterial as Differentiator

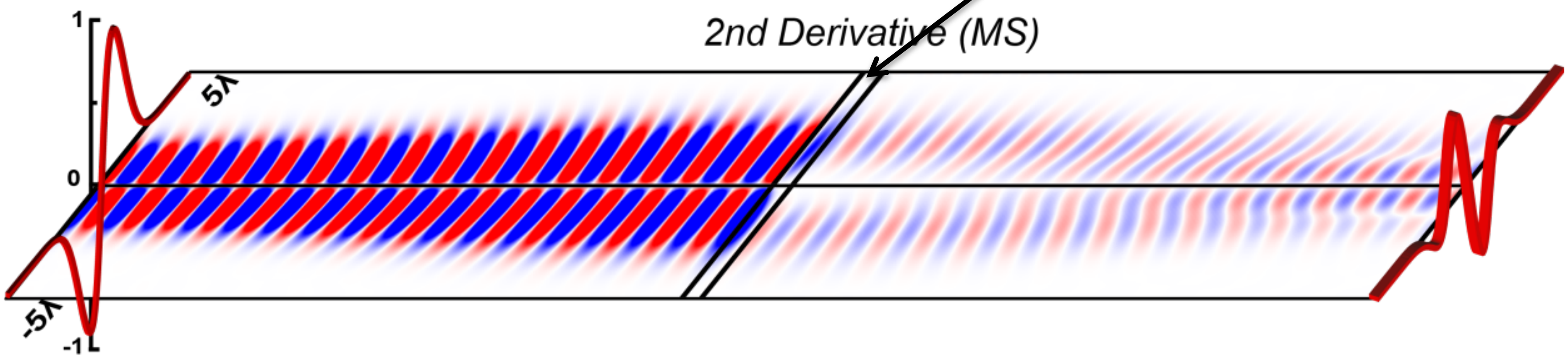


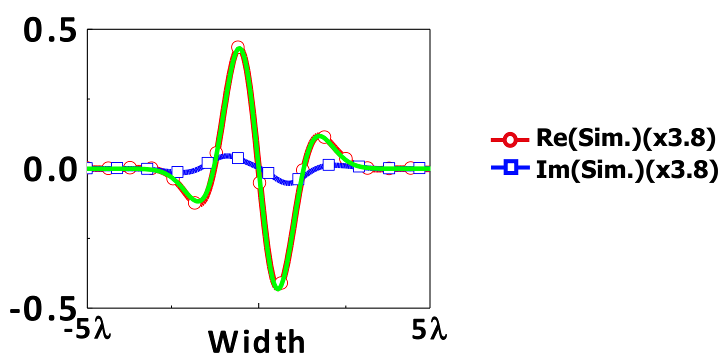


A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

Metamaterial as 2^{nd} Differentiator

$$\varepsilon_{ms}(y)/\varepsilon_o = \mu_{ms}(y)/\mu_o = i2\left[\lambda_o/(2\pi\Delta)\right]\ln\left(-iW/(2y)\right)$$



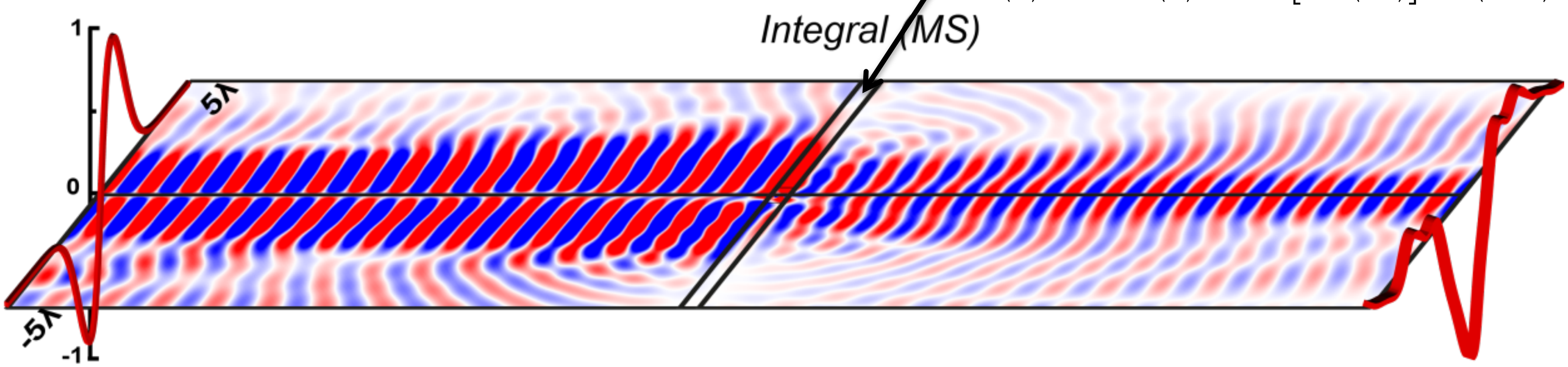


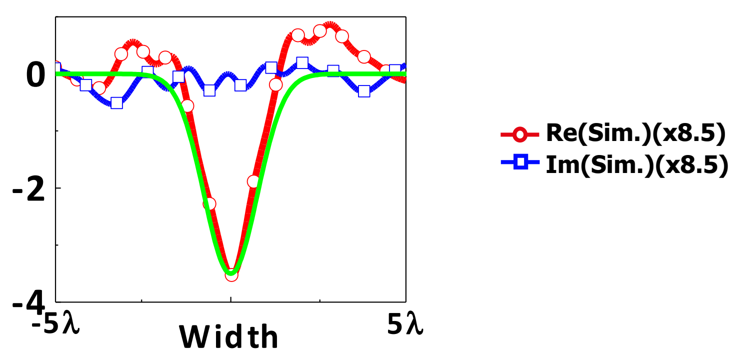
A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

Metamaterial as Integrator

$$\varepsilon_{ms}(y)/\varepsilon_o = \mu_{ms}(y)/\mu_o = i\left[\lambda_o/(2\pi\Delta)\right]\ln(iy/d)$$

$$\varepsilon_{ms}(y)/\varepsilon_o = \mu_{ms}(y)/\mu_o = -\left[\lambda_o/(4\Delta)\right]sign(y/d)$$

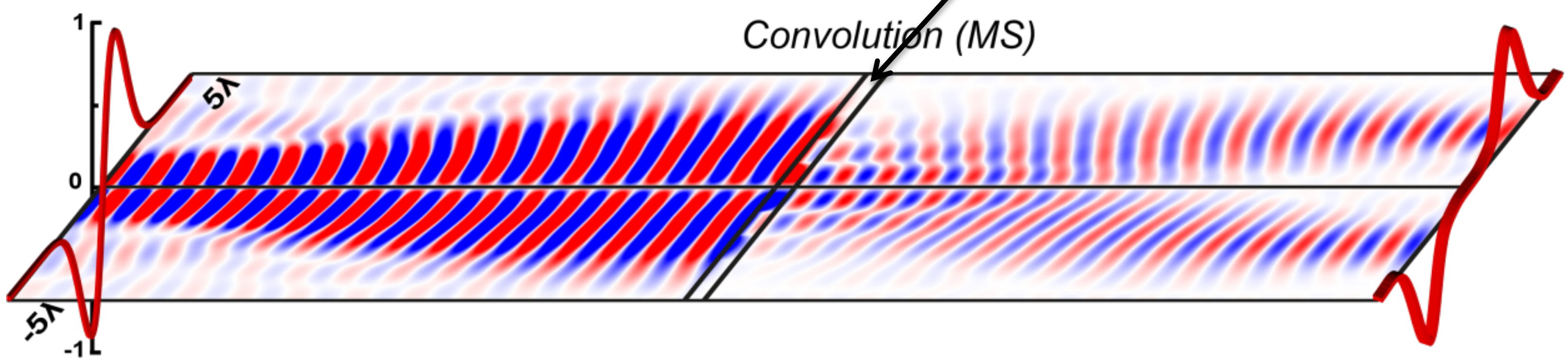


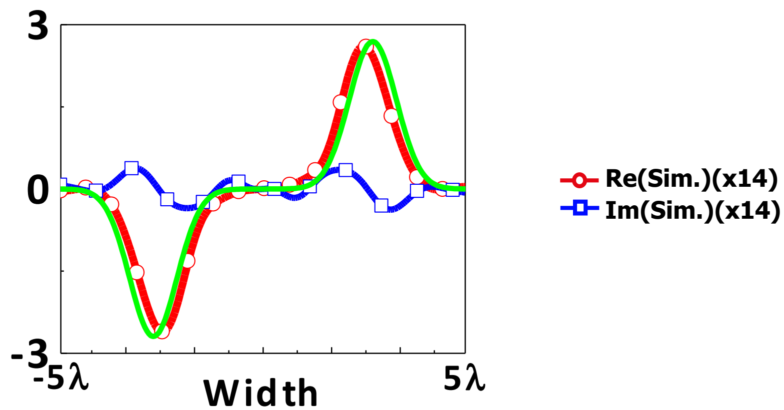


A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

Metamaterial as Convolver

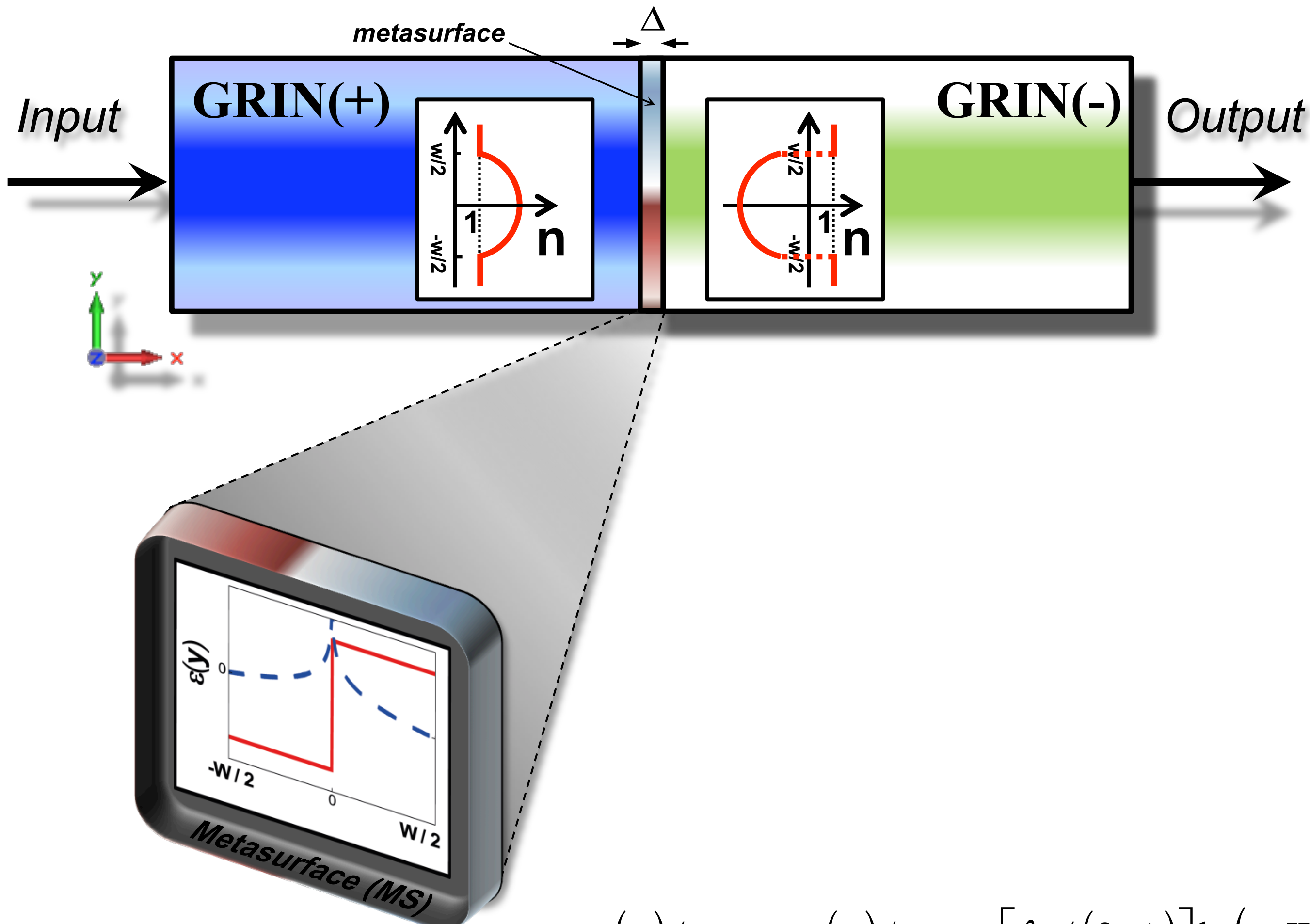
$$\varepsilon_{ms}(y)/\varepsilon_o = \mu_{ms}(y)/\mu_o = i\left[\lambda_o/(2\pi\Delta)\right]\ln\left[i/sinc\left(W_k y/(2s^2)\right)\right]$$





A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

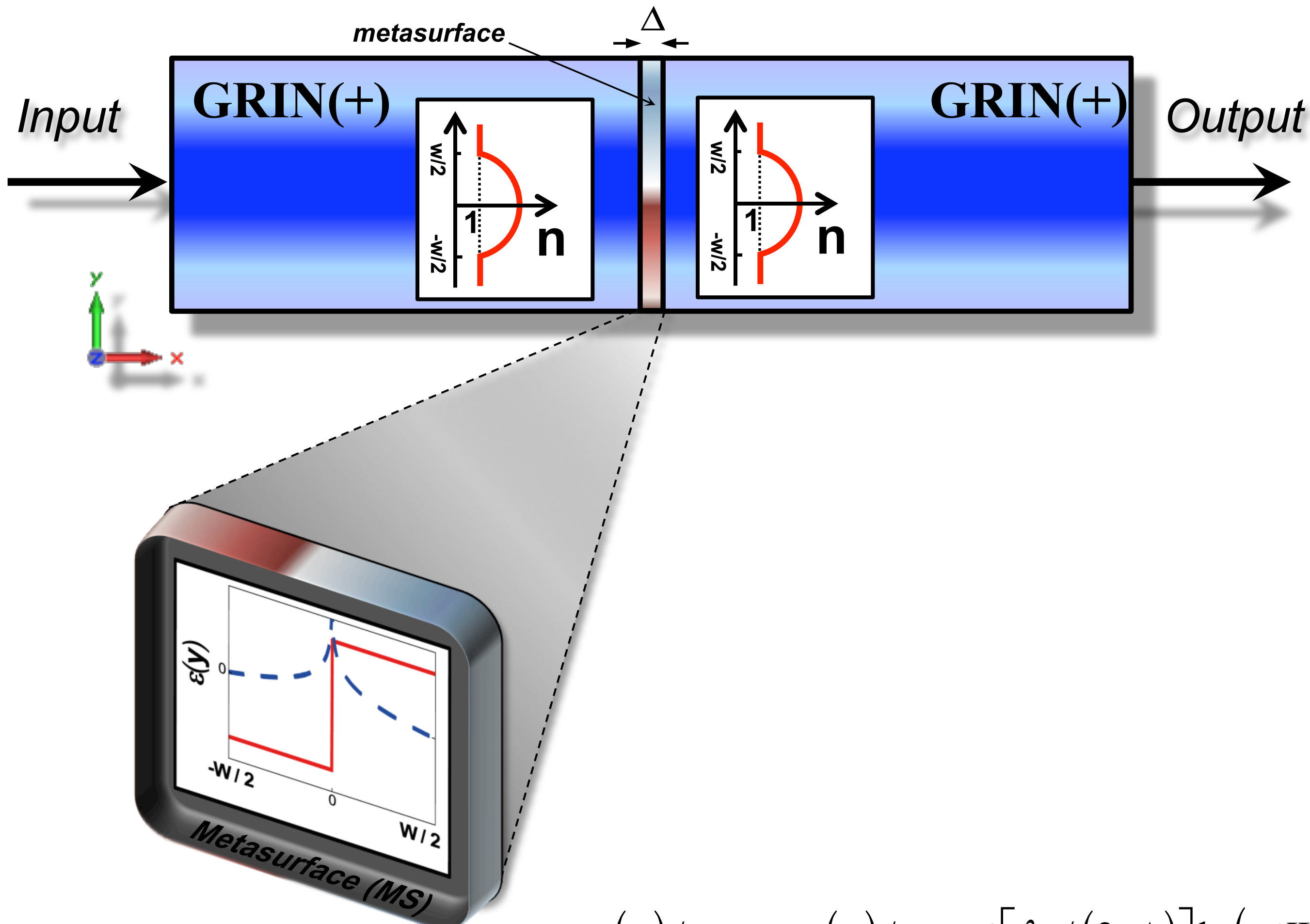
Realistic Materials for Structures



$$\varepsilon_{ms}(y)/\varepsilon_o = \mu_{ms}(y)/\mu_o = i\left[\lambda_o/(2\pi\Delta)\right]\ln\left(-iW/(2y)\right)$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

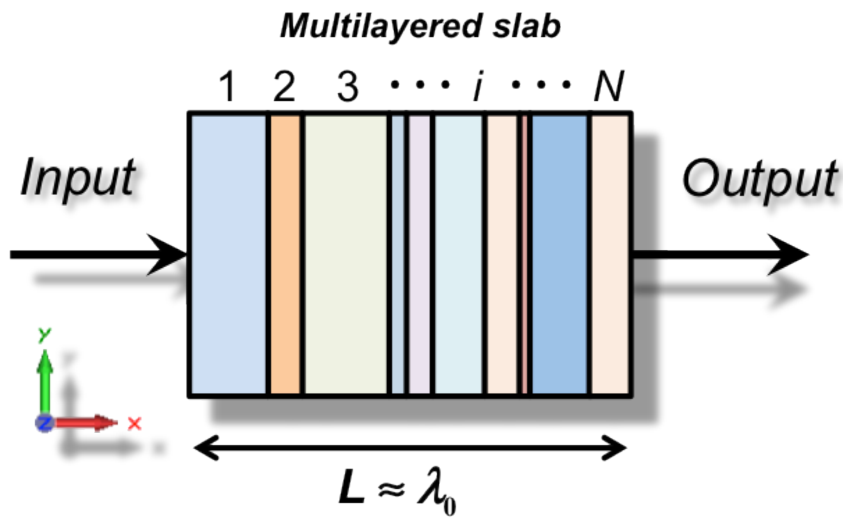
Realistic Materials for Structures



$$\varepsilon_{ms}(y)/\varepsilon_o = \mu_{ms}(y)/\mu_o = i\left[\lambda_o/(2\pi\Delta)\right]\ln\left(-iW/(2y)\right)$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

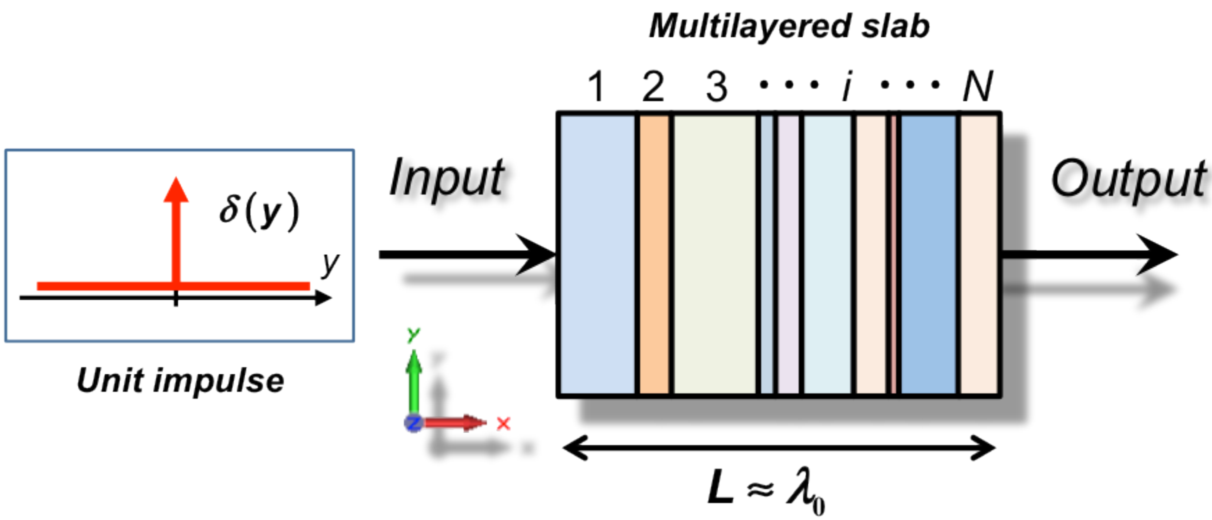
Green's Function Approach



$$g(y) = \int f(y')G(y-y')\,dy'$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

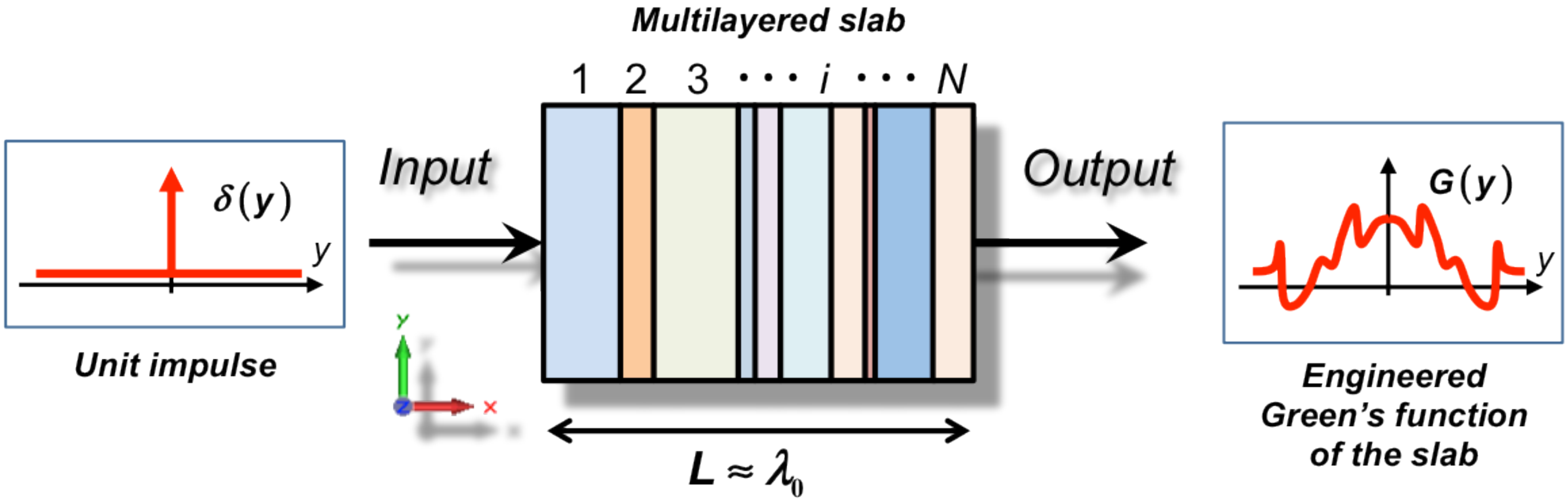
Green's Function Approach



$$g(y) = \int f(y')G(y-y')\,dy'$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

Green's Function Approach

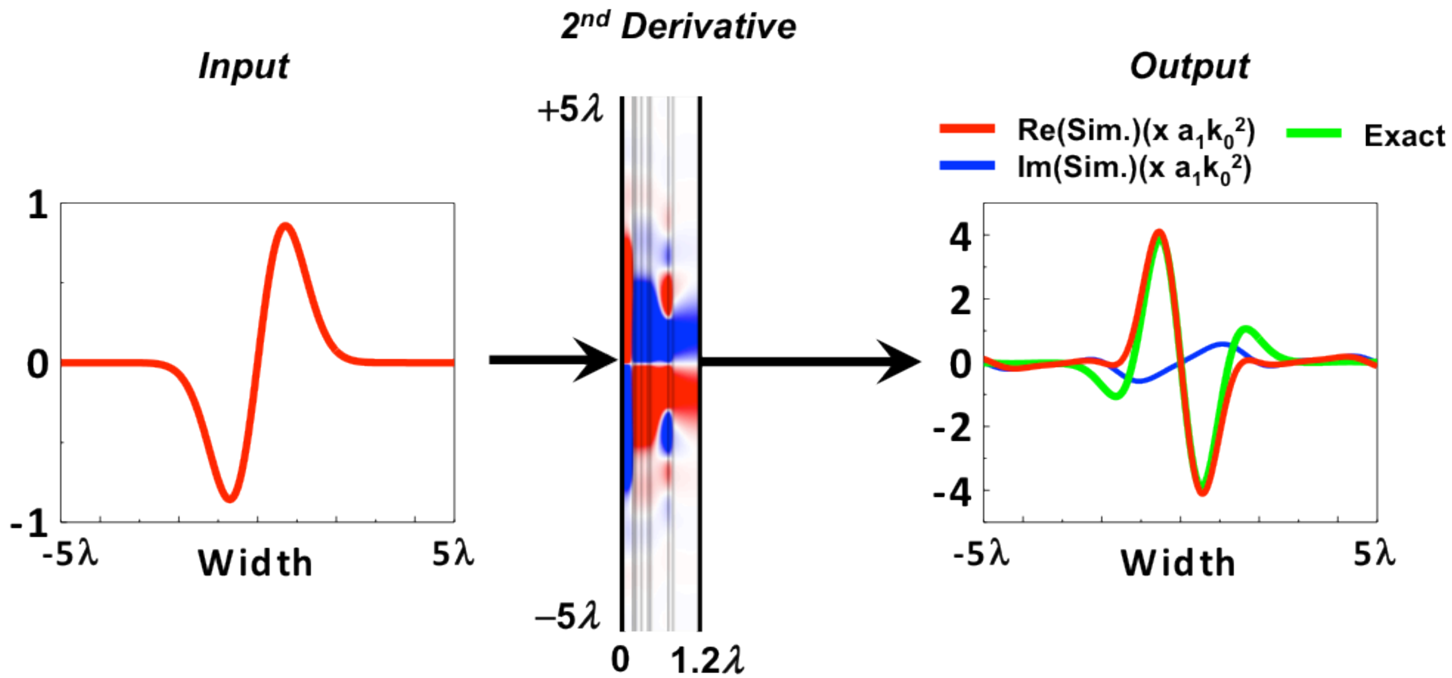


$$g(y) = \int f(y')G(y-y')\,dy'$$

$$\frac{d^2 f(y)}{dy^2} \propto \int f(y')\delta^{(2)}(y-y')\,dy'$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

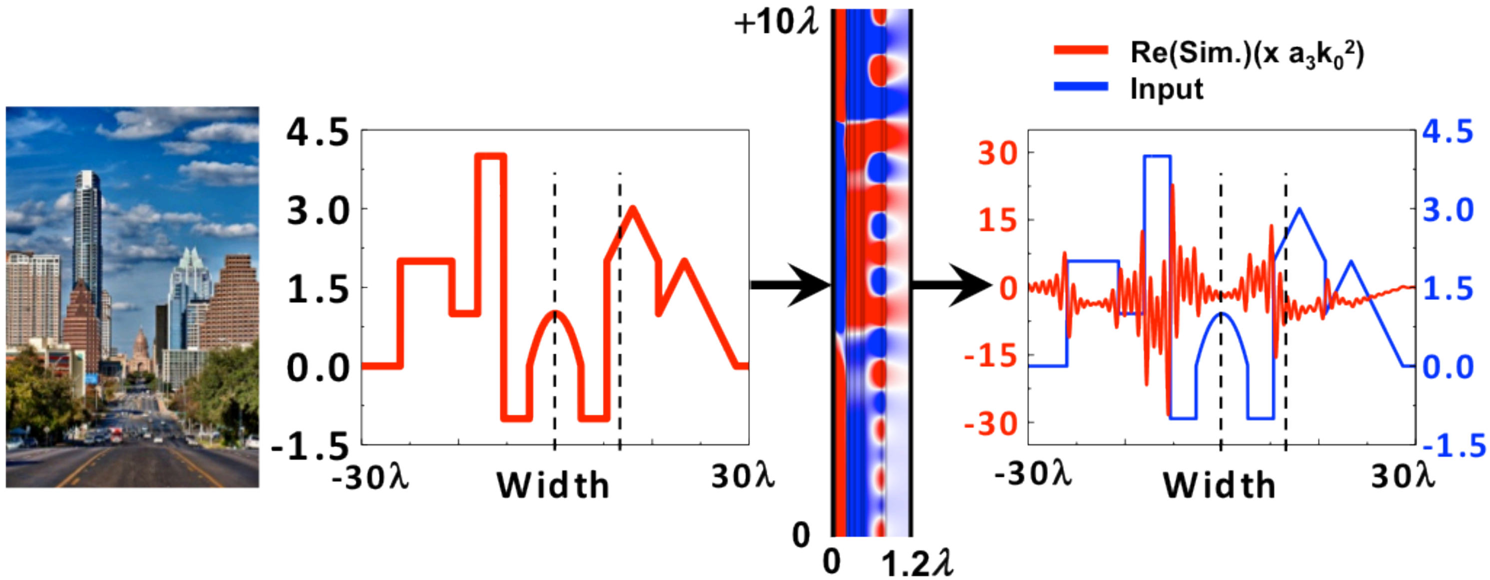
2^{nd} Differentiation: Green's Function Approach



	1	2	3	4	5	6	7	8	9	10
ε_r	13.85	5.98	4.44	0.06	0.03	0.01	-0.003	-2.12	2.30	0.08
d	$\lambda_0/293.4$	$\lambda_0/6.0$	$\lambda_0/212.9$	$\lambda_0/24.2$	$\lambda_0/12.1$	$\lambda_0/9.8$	$\lambda_0/25.0$	$\lambda_0/3.6$	$\lambda_0/14.5$	$\lambda_0/2.4$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

2^{nd} Differentiation: Green's Function Approach



A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014

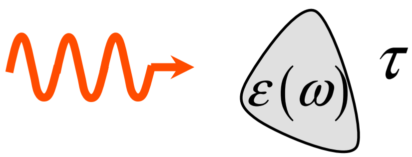
Fields and Waves in Metamaterials
Part 2

Nader Engheta

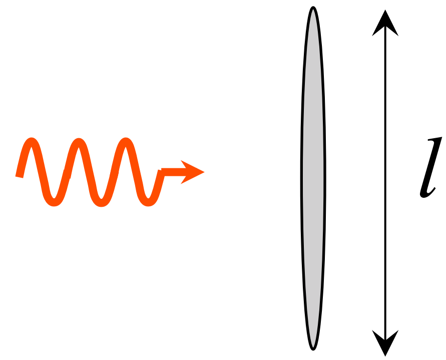
University of Pennsylvania
Philadelphia, PA 19104, USA

August 16-17, 2014

Light-Matter Interaction



$$\omega \leftrightarrow 2\pi / T$$



$$k \leftrightarrow 2\pi / \lambda$$

$$k \equiv \frac{2\pi}{\lambda} = \omega\sqrt{\varepsilon\mu}$$

Metamaterials

N. Engheta, <u>Science,</u> 340, 286 (2013)

What will happen, if epsilon is near zero?

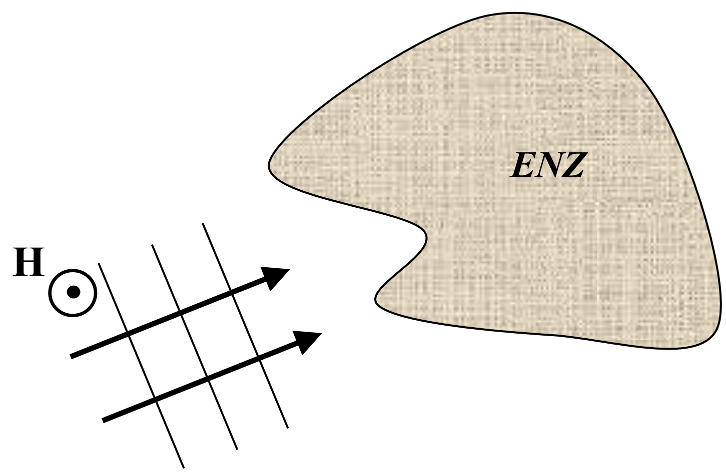
- *Maxwell Equations* $\nabla \times \boldsymbol{H} = -i\omega\varepsilon \boldsymbol{E} \longrightarrow \nabla \times \boldsymbol{H} = 0$

  $\nabla \times \boldsymbol{E} = i\omega\mu \boldsymbol{H}$

- *2-D Scenario with TM polarization*

$$\mathbf{H} = H(x,y)\ \hat{\mathbf{u}}_z$$

$$\mathbf{E} = \frac{1}{-i\omega\varepsilon}\nabla H(x,y) \times \hat{\mathbf{u}}_z$$



$H = const.$ *inside ENZ material.* $n = \sqrt{\varepsilon\mu} \to 0$

M. Silveirinha & N. Engheta, Phys. Rev. Lett. 97, 157403, Oct 2006

What will happen, if epsilon is near zero?

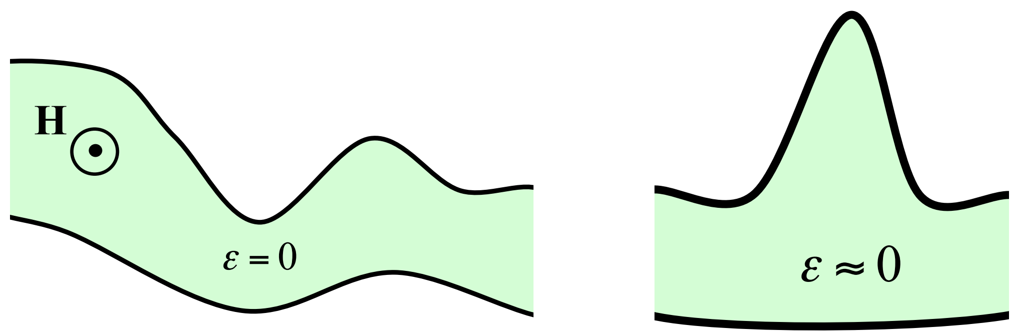
- **Maxwell Equations** $\nabla \times \boldsymbol{H} = -i\omega\varepsilon \boldsymbol{E}$ → $\nabla \times \boldsymbol{H} = 0$

  $\nabla \times \boldsymbol{E} = i\omega\mu \boldsymbol{H}$

- **2-D Scenario with TM polarization**

$$\mathbf{H} = H(x,y)\ \hat{\mathbf{u}}_z$$

$$\mathbf{E} = \frac{1}{-i\omega\varepsilon}\nabla H(x,y)\times\hat{\mathbf{u}}$$



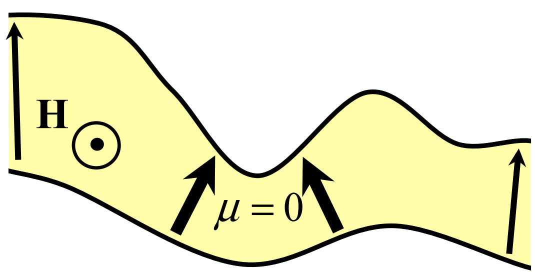
$H = const.$ ***inside ENZ material.*** $n = \sqrt{\varepsilon\mu} \rightarrow 0$

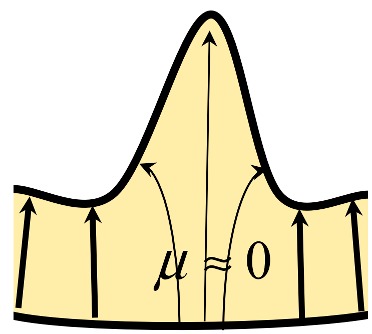
What will happen, if mu is near zero?

- *Maxwell Equations* $\nabla \times \boldsymbol{H} = -i\omega\varepsilon \boldsymbol{E}$

  $\nabla \times \boldsymbol{E} = i\omega\mu \boldsymbol{H}$ → $\nabla \times \boldsymbol{E} = 0$

- *2-D Scenario with TM polarization*





"ENZ Supercoupling"

M. Silveirinha & N. Engheta, Phys. Rev. Lett. 97, 157403, Oct 2006

M. Silveirinha & N. Engheta, Phys. Rev. B., 76, 245109 (2007)

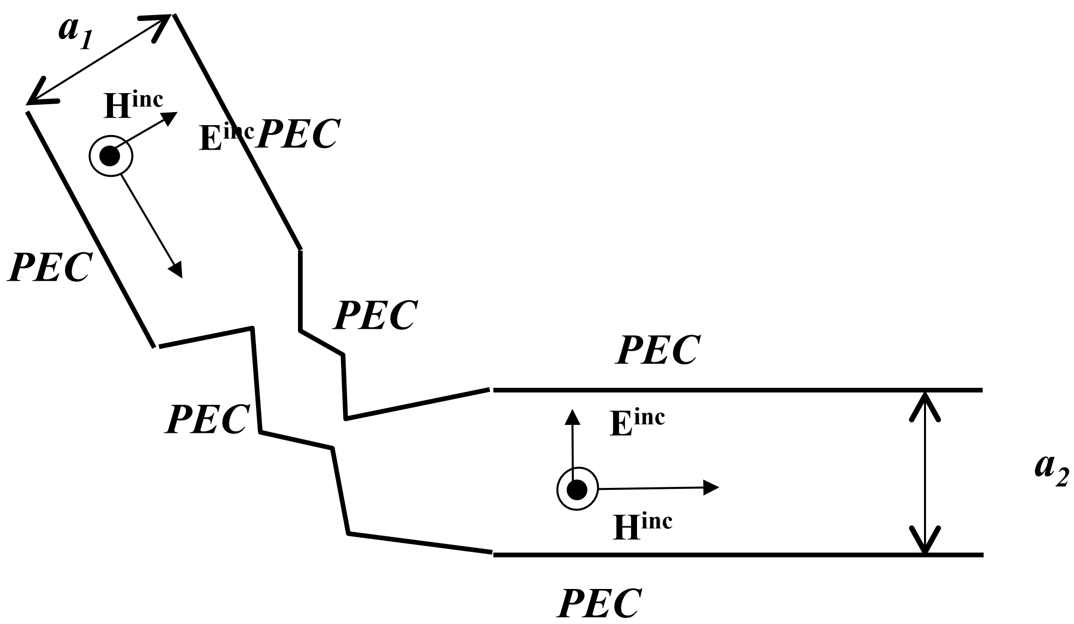
B. Edwards, A. Alu, M. Young, M. Silveirinha, N. Engheta, Phys. Rev. Lett., 100, 033903, 245109 (2008)

A. Alu, M. Silveirinha, N. Engheta, Phys. Rev. E., 78, 016604 (2008)

A. Alu, N. Engheta, Phys. Rev. B., 78, 045102 (2008)

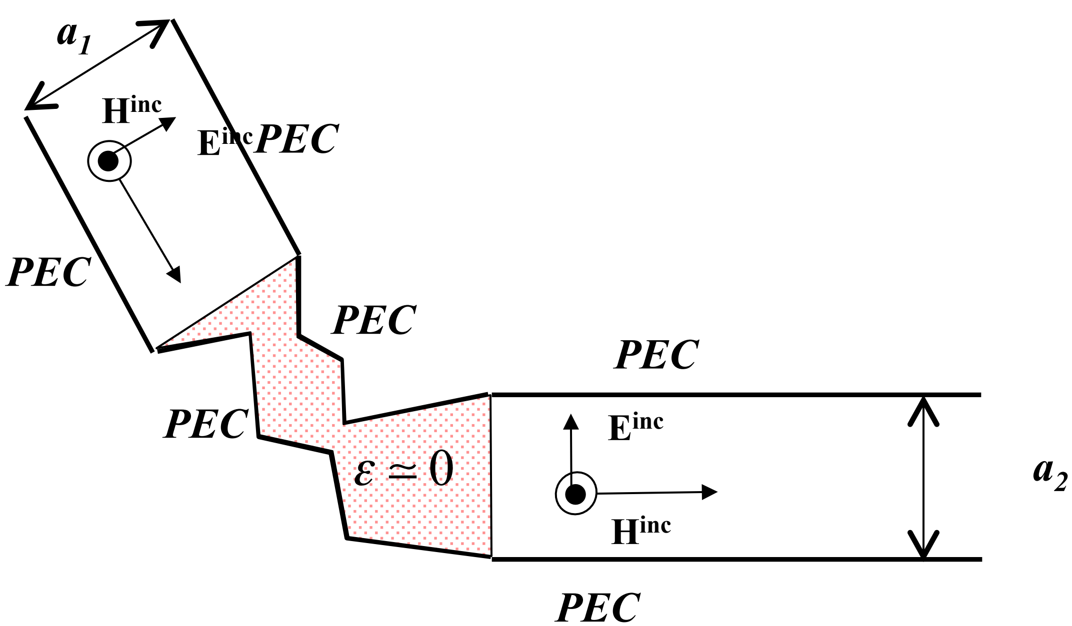
A. Alu, N. Engheta, Phys. Rev. B., 78, 035440 (2008)

"Supercoupling" in Sub-λ Channels



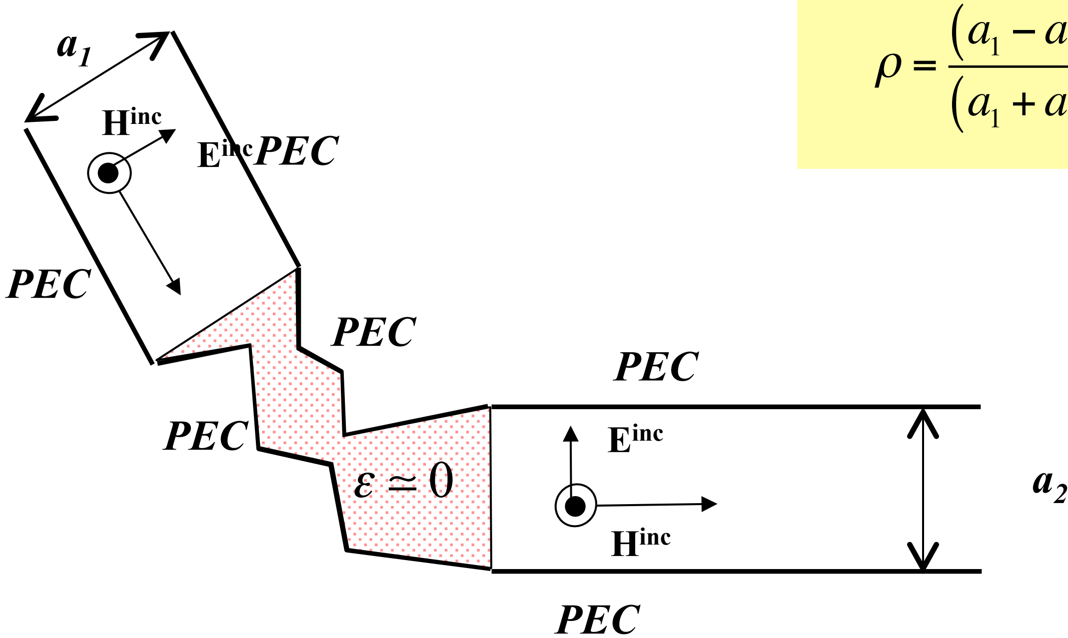
M. Silveirinha & N. Engheta, Phys. Rev. Lett. 97, 157403, Oct 2006

"Supercoupling" in Sub-λ Channels



M. Silveirinha & N. Engheta, Phys. Rev. Lett. 97, 157403, Oct 2006

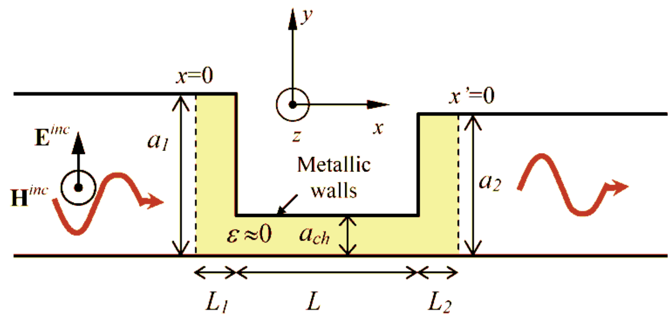
"Supercoupling" in Sub-λ Channels

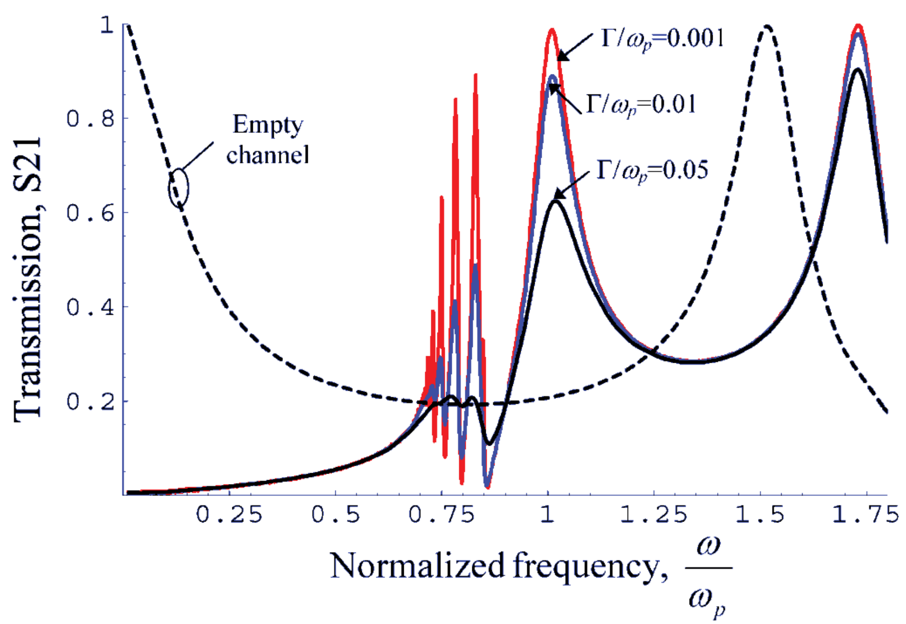


$$\rho = \frac{(a_1 - a_2) + ik_0\mu_r A_D}{(a_1 + a_2) - ik_0\mu_r A_D}$$

M. Silveirinha & N. Engheta,
<u>Phys. Rev. Lett.</u> 97, 157403, Oct 2006

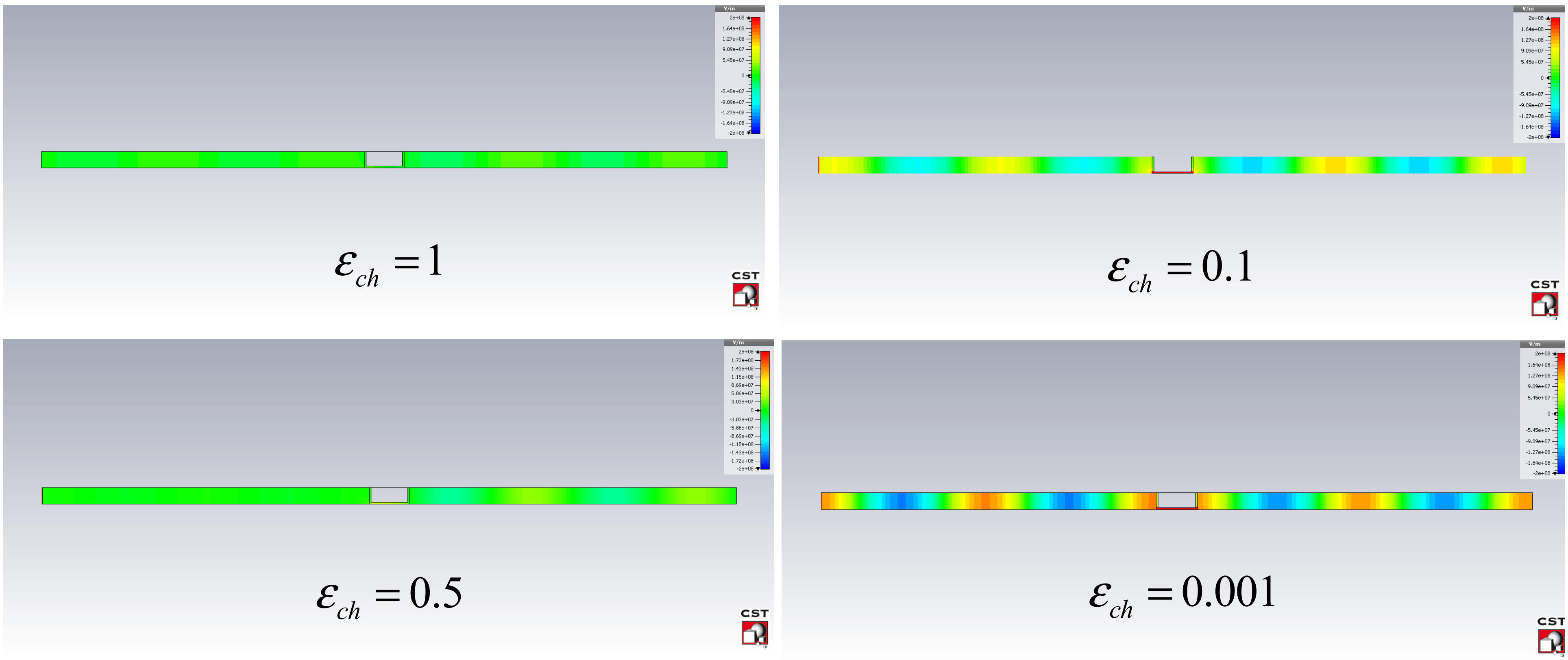
U-shaped Waveguide Transition & Supercoupling (cont'd)



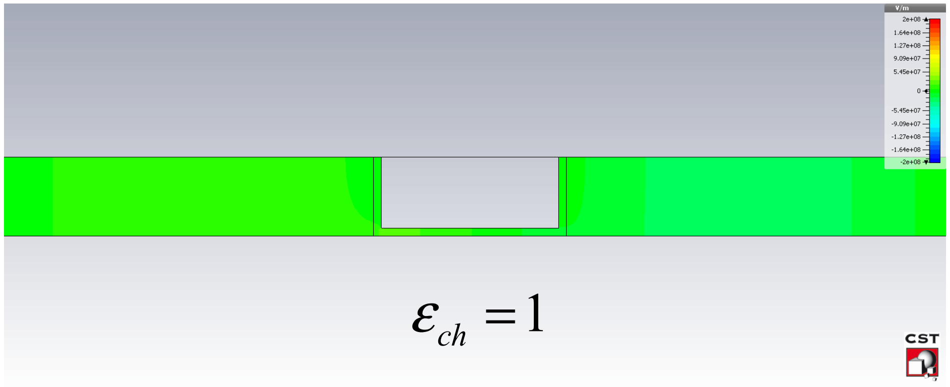


M. Silveirinha & N. Engheta, Phys. Rev. B., 76, 245109 (2007)

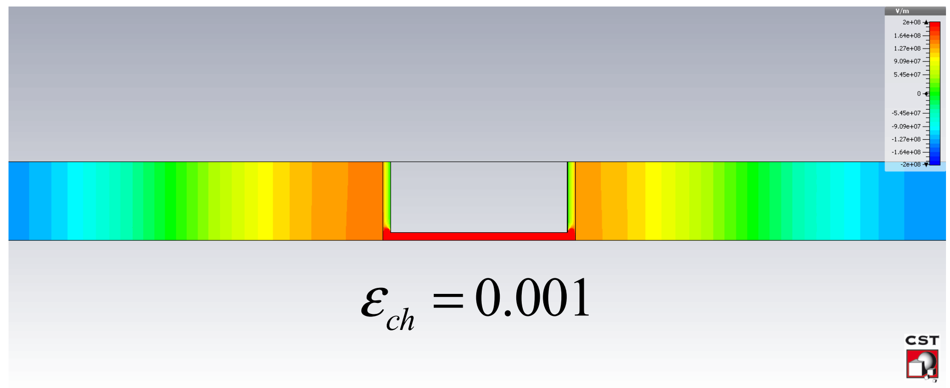
Simulation Results: 2D scenario



Simulation Results: 2D scenario

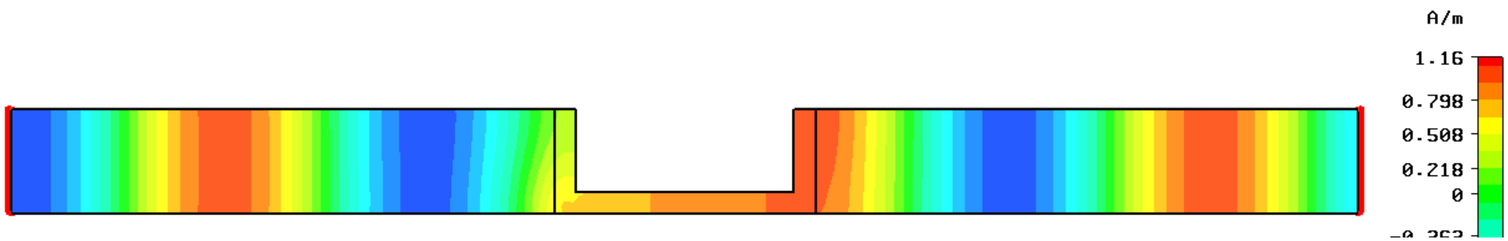


$$\varepsilon_{ch} = 1$$



$$\varepsilon_{ch} = 0.001$$

Intuitive Explanation



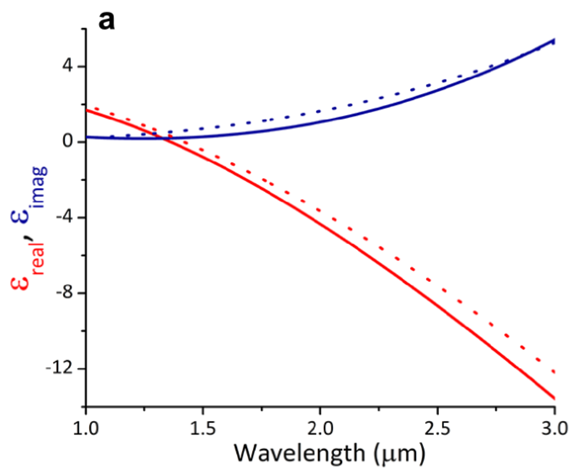
$$Z_{wg} = \frac{V}{I} = \frac{E \cdot h}{H \cdot W} = \frac{E}{H}\frac{h}{W} = \sqrt{\frac{\mu}{\varepsilon}}\frac{h}{W}$$

$$Z_{wg1} = Z_{wg2}$$

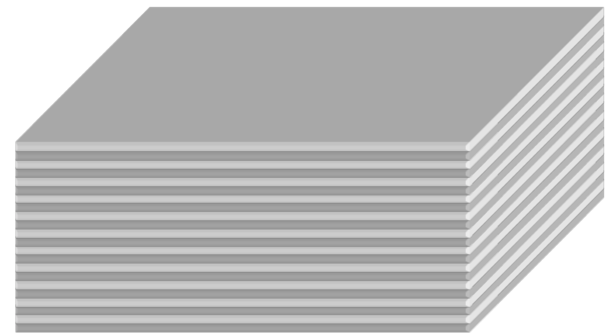
$$\sqrt{\frac{\mu_1}{\varepsilon_1}}\frac{h_1}{W_1} = \sqrt{\frac{\mu_2}{\varepsilon_2}}\frac{h_2}{W_2}$$

B. Edwards, A. Alu, M. Young, M. Silveirinha, N. Engheta, Phys. Rev. Lett., 100, 033903, 245109 (2008)

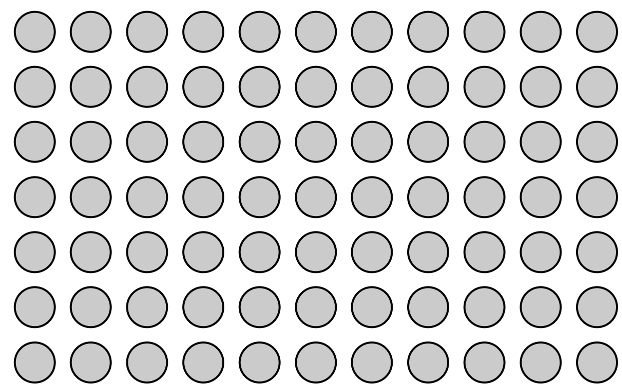
ENZ Structures



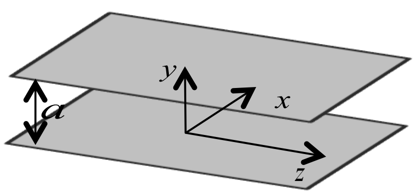
ITO



$\mathrm{Re}(\varepsilon) \cong 0$

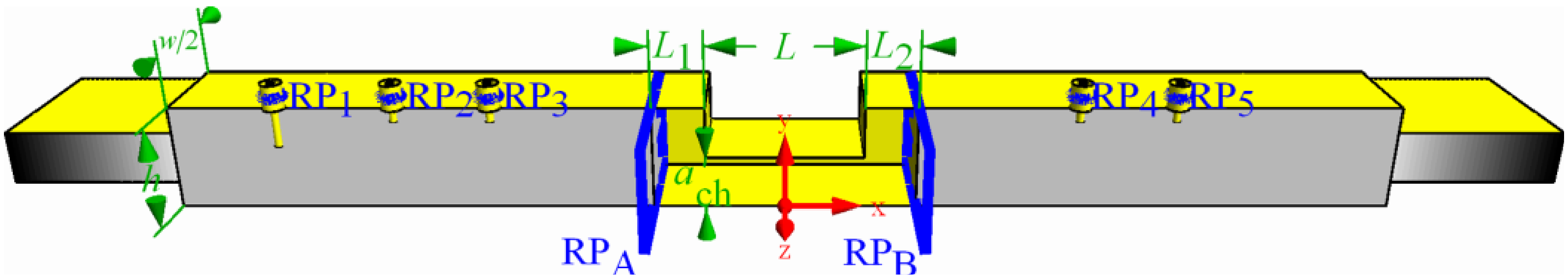


$\mathrm{Re}(\varepsilon) \cong 0$



$$k_z = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r - \frac{1}{\omega^2\mu_o\varepsilon_o}\left(\frac{\pi}{a}\right)^2}$$

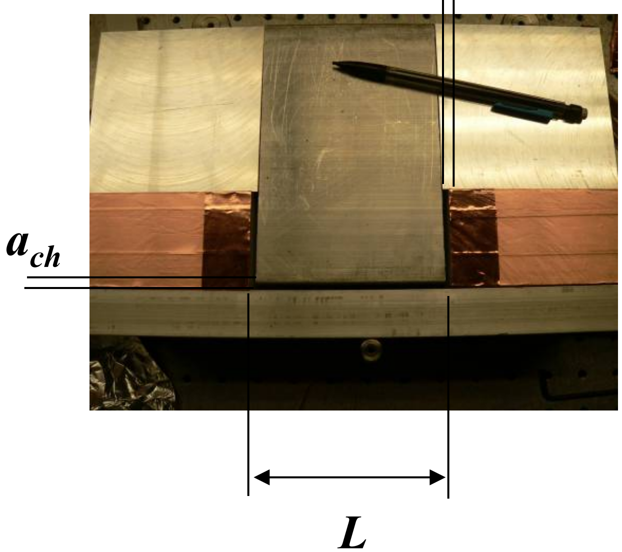
Experimental Verification

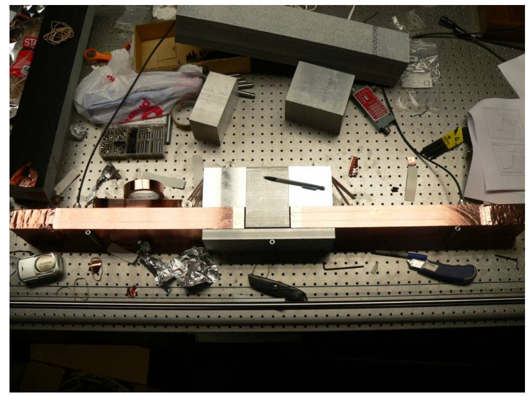


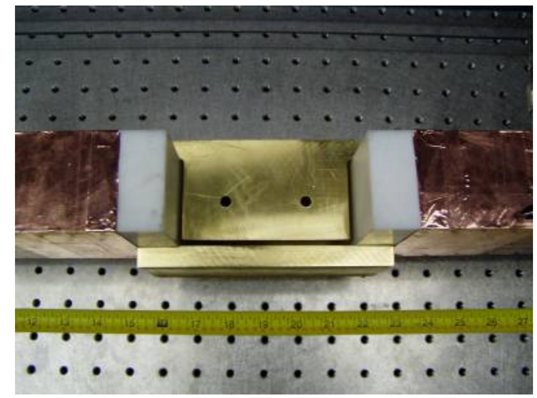
$$\beta_z = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\varepsilon_r - c^2/\left(4f^2w^2\right)} \longrightarrow \varepsilon_{eff}/\varepsilon_0 = \varepsilon_r - c^2/\left(4f^2w^2\right)$$

B. Edwards, A. Alu, M. Young, M. Silveirinha, N. Engheta, Phys. Rev. Lett., 100, 033903, 245109 (2008)

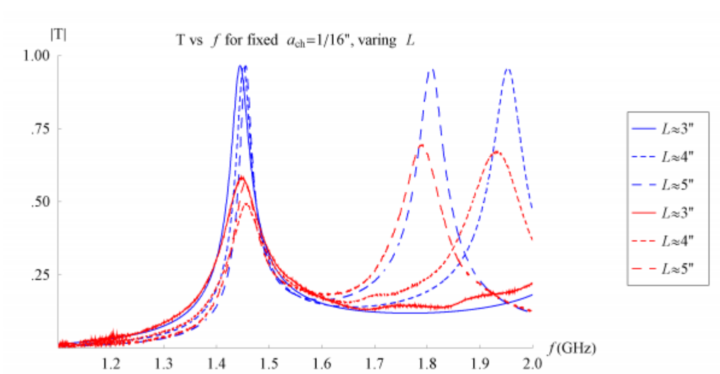
Experimental Verification

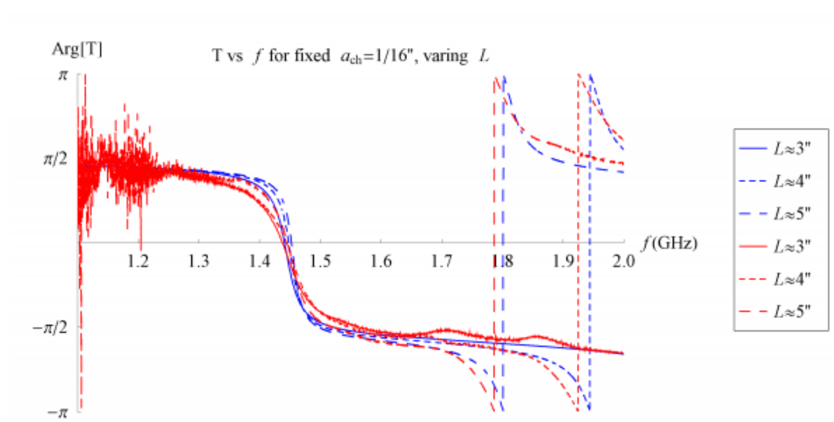




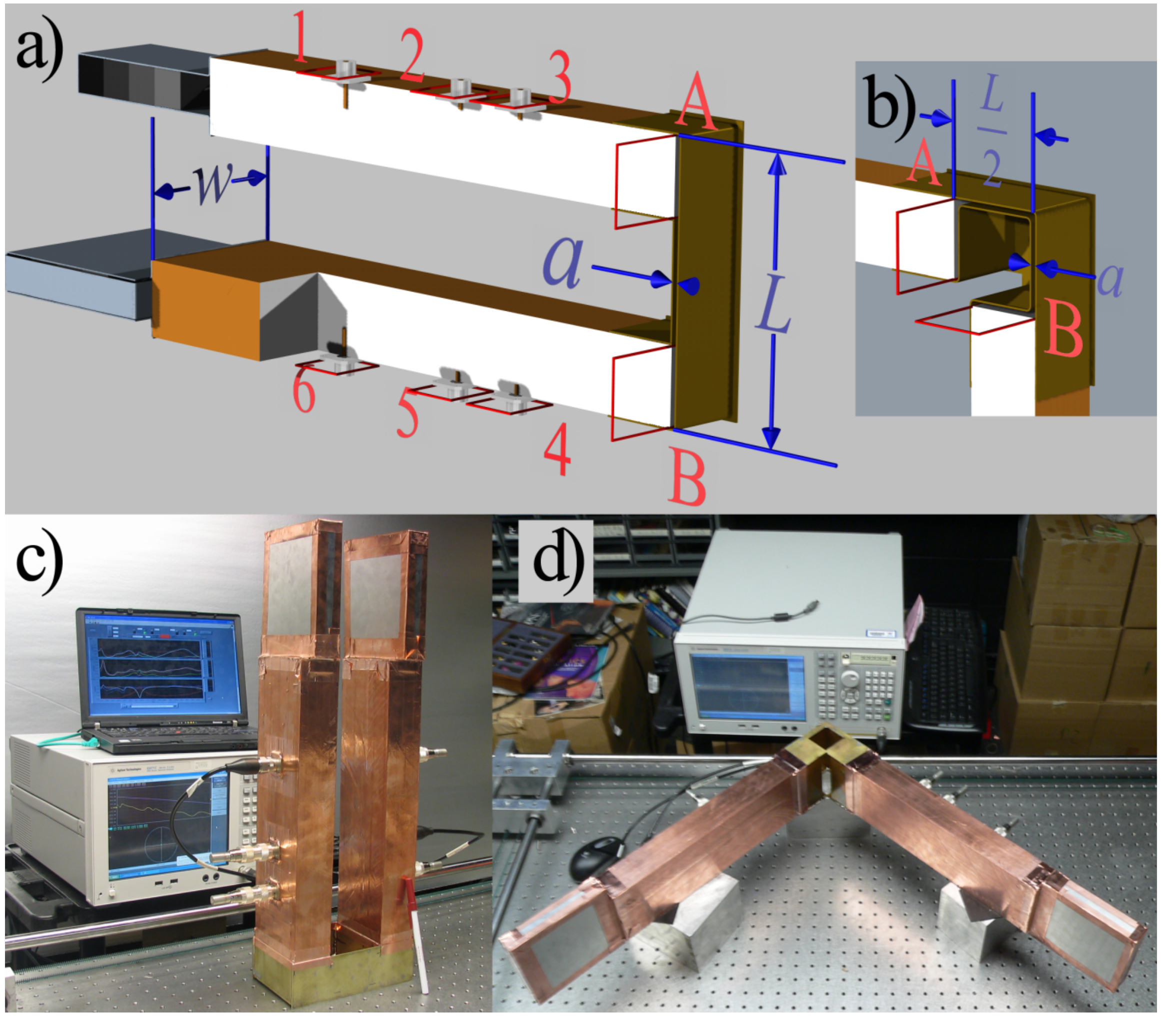


B. Edwards, A. Alù, M. Young, M. Silveirinha, N. Engheta
Phys. Rev. Lett. 100, 033103 (2008)

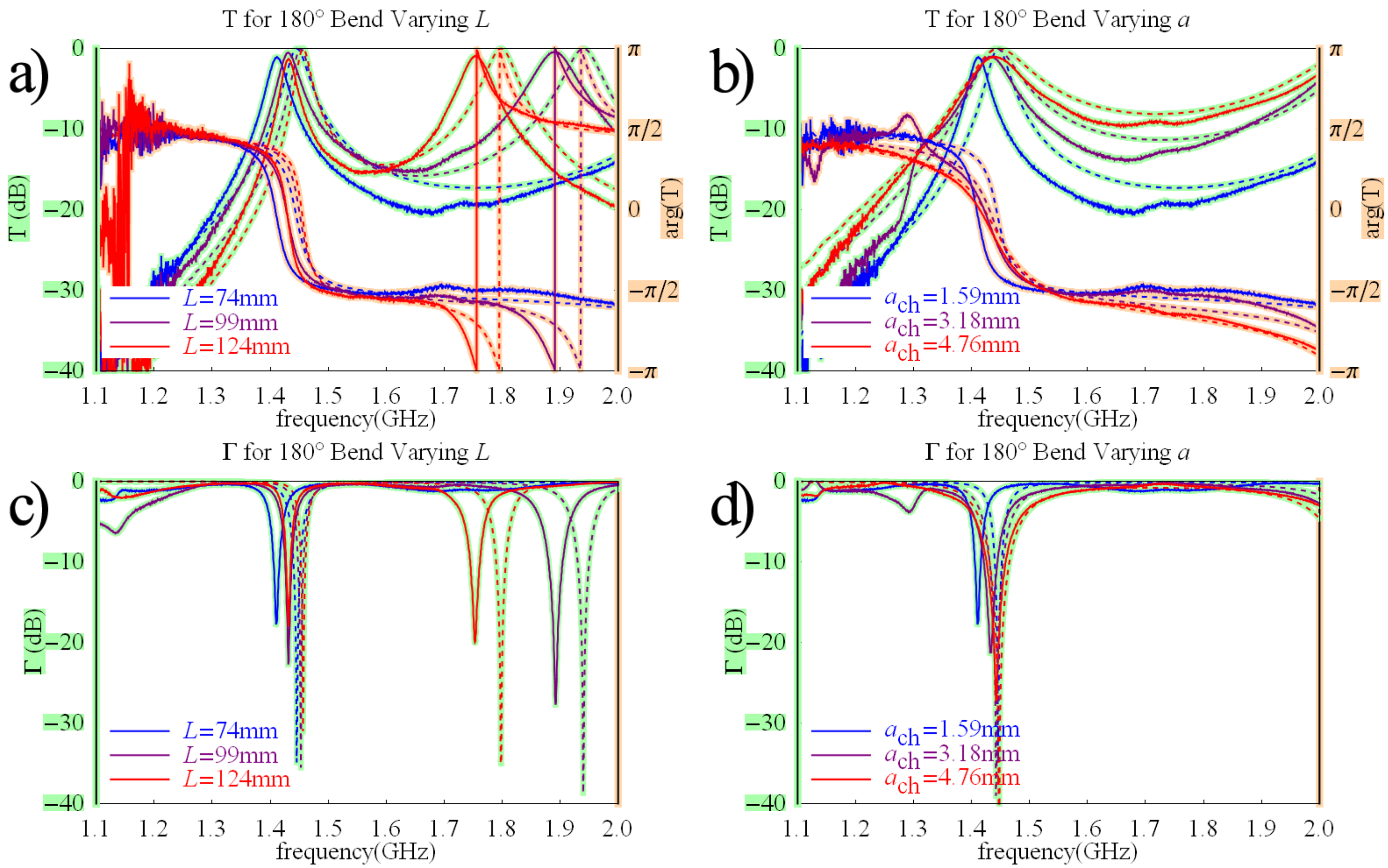




Waveguide Bends with Narrow Channels

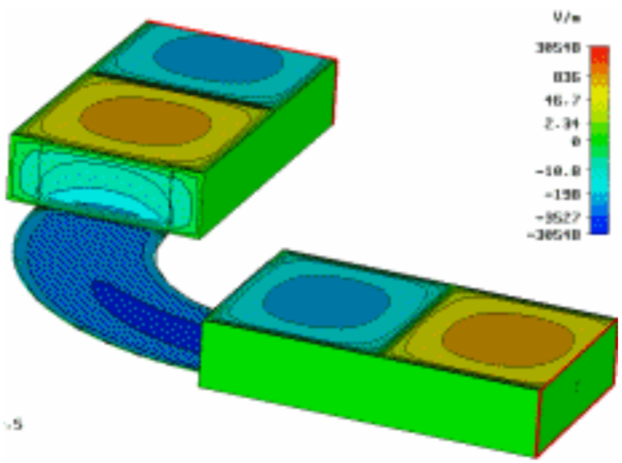


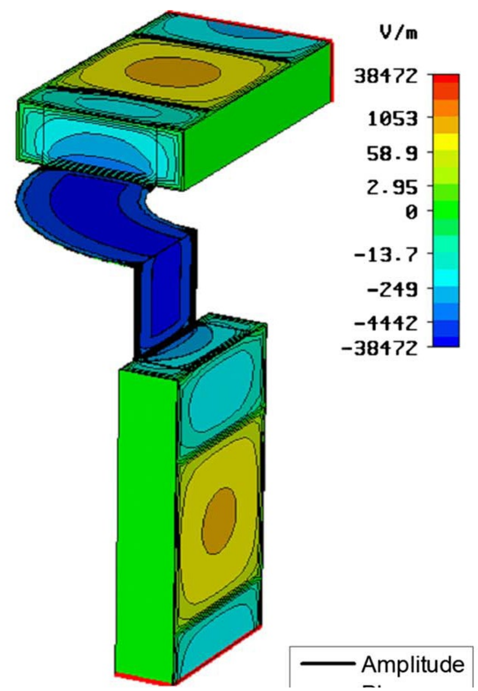
180-degree Waveguide Bends



B. Edwards, A. Alù, M. Silveirinha, N. Engheta
Journal of Applied Physics, 2009

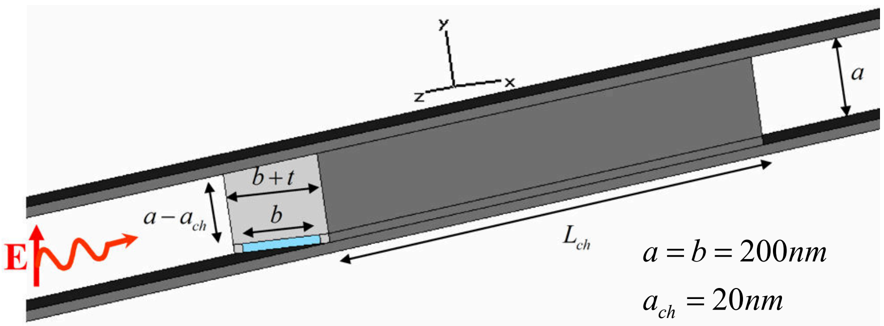
Waveguide Bends with Narrow Channels





A. Alu, M. Silveirinha, N. Engheta, Phys. Rev. E., 78, 016604 (2008)

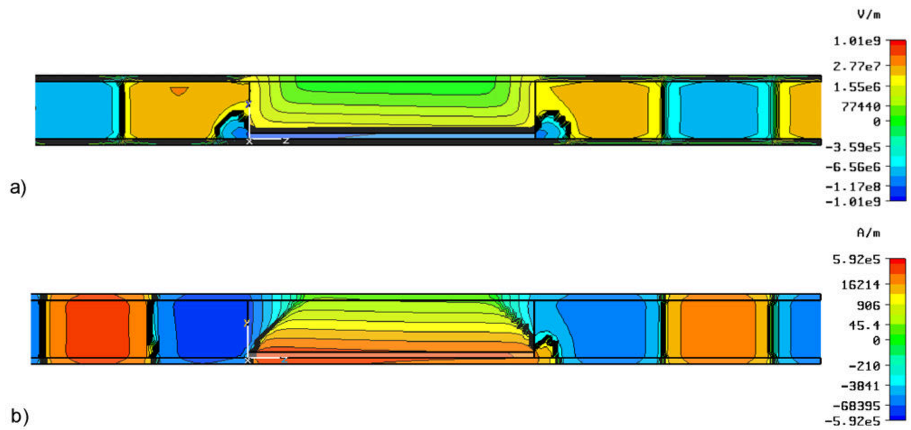
Plasmonic Channels and ENZ Tunneling

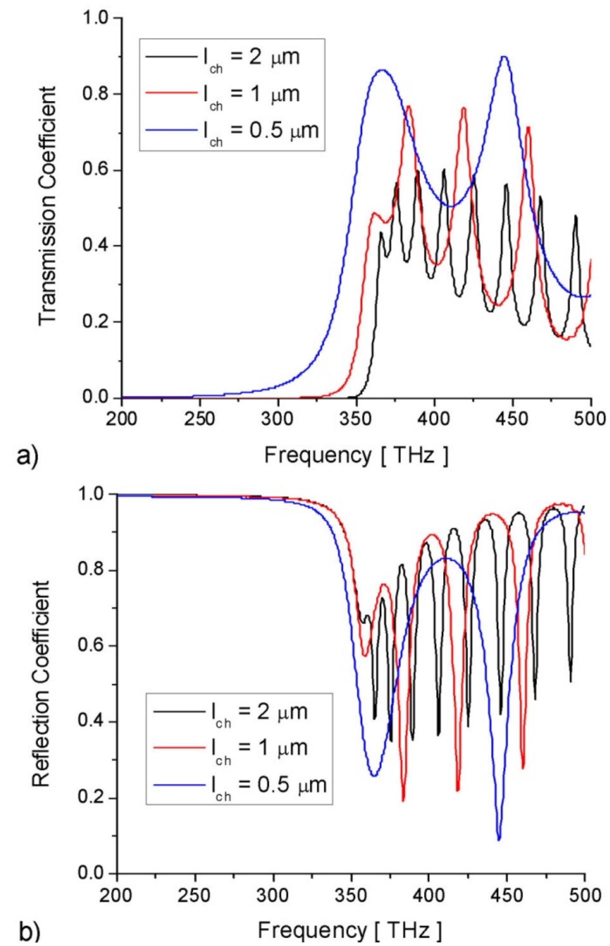


$a = b = 200nm$

$a_{ch} = 20nm$

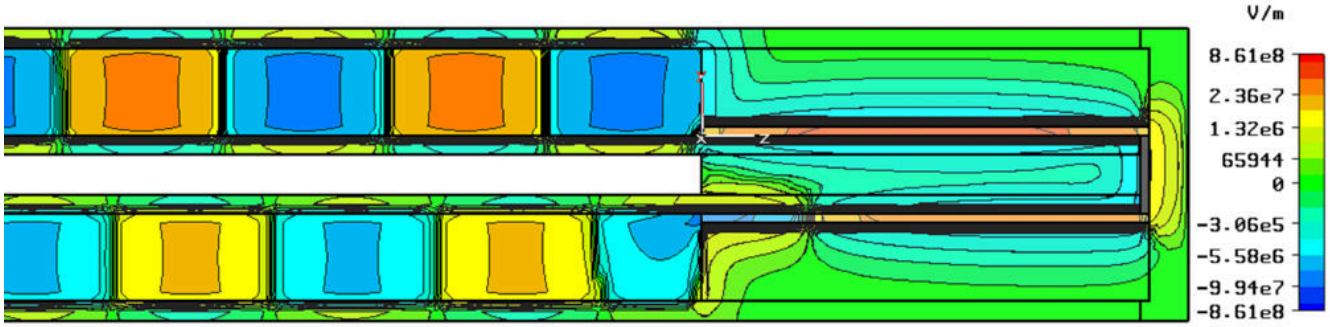
$t = 50nm$

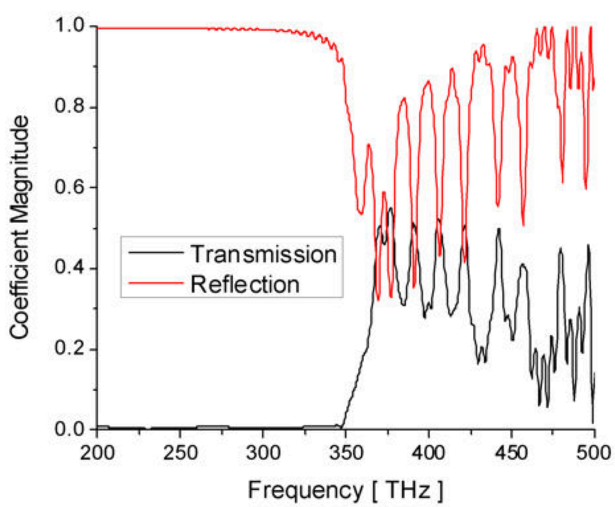




A. Alù and N. Engheta
Phys. Rev. B, 78, 2008

Plasmonic Channels and ENZ Tunneling





$a = b = 200nm$

$a_{ch} = 20nm$

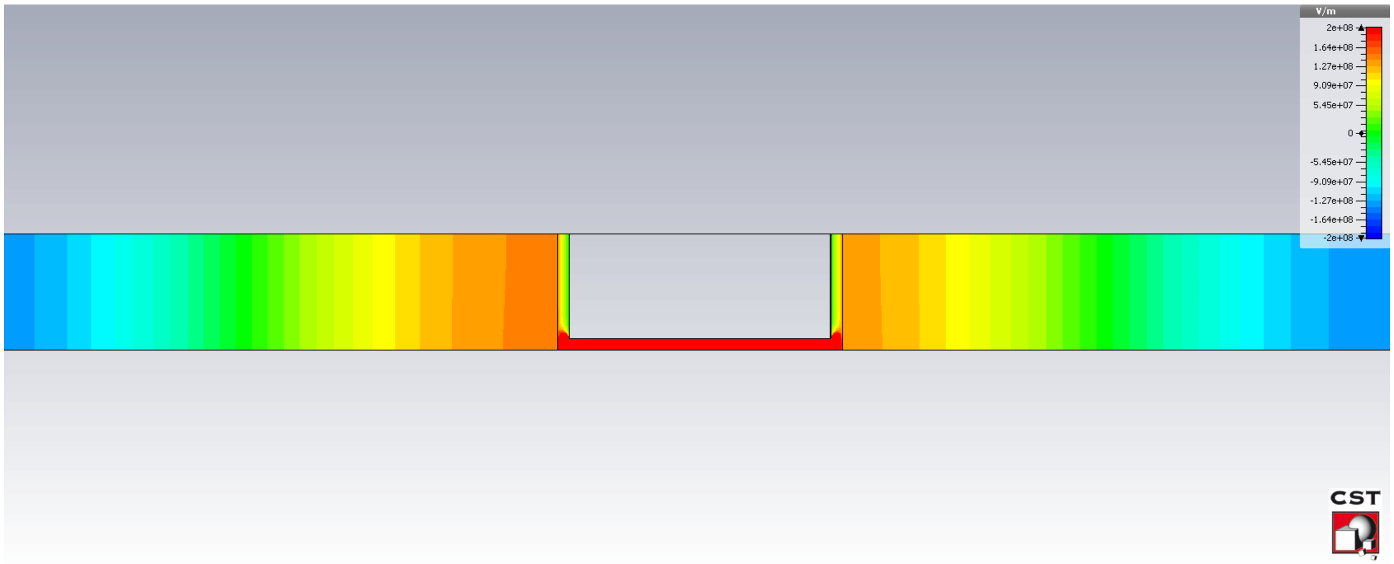
$t = 50nm$

A. Alù and N. Engheta
Phys. Rev. B. 78, 2008

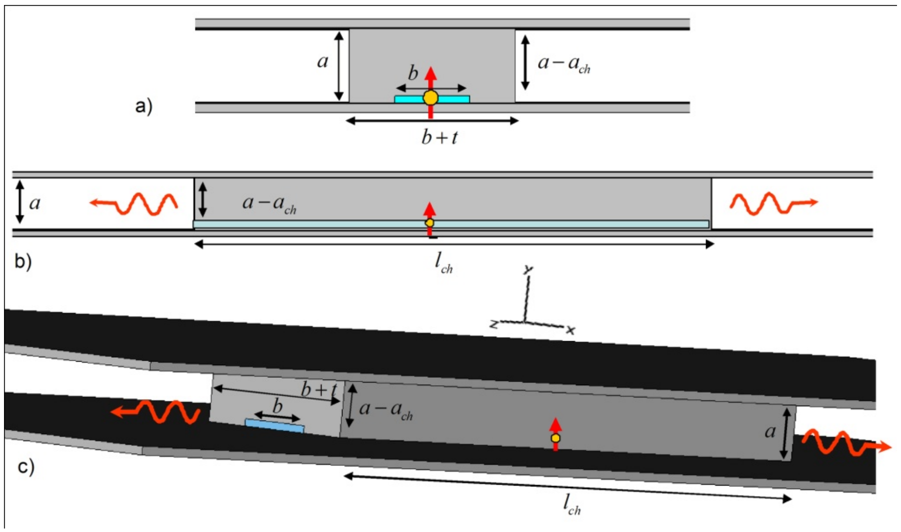
ENZ and Spontaneous Emission Rate of Optical Emitters

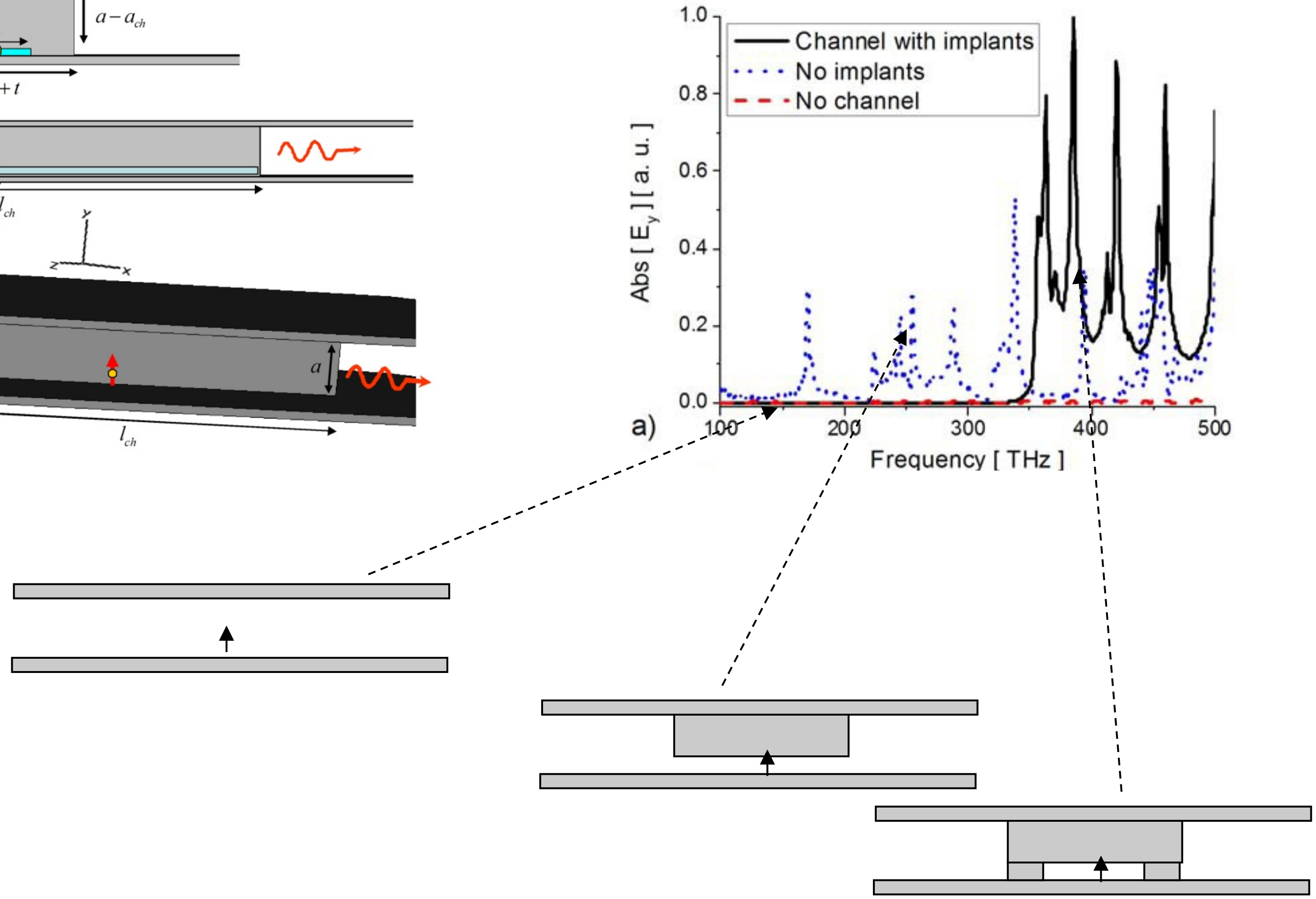
A. Alu and N. Engheta, Phys. Rev. Lett. 103, 043902 (2009)

Field Enhancement Using ENZ



Enhancement of Optical Emitters





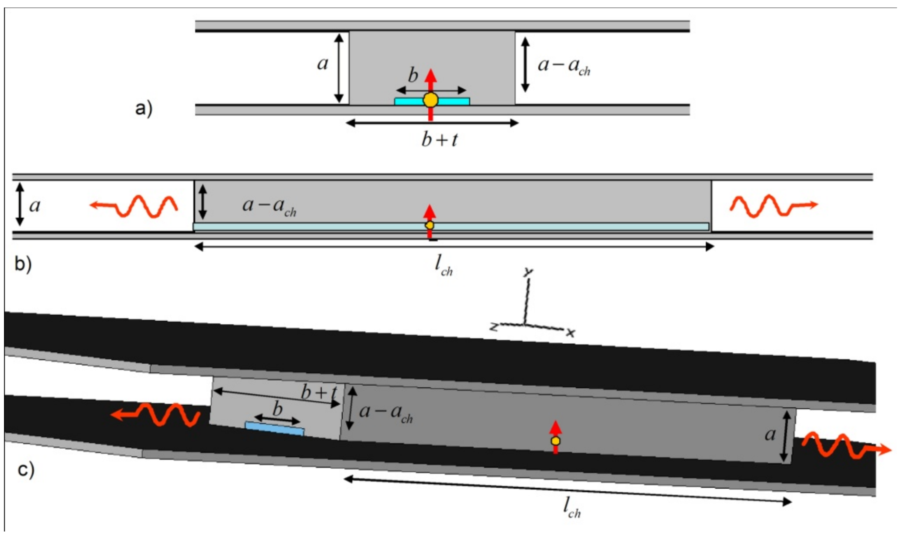
$a = b = 200nm$

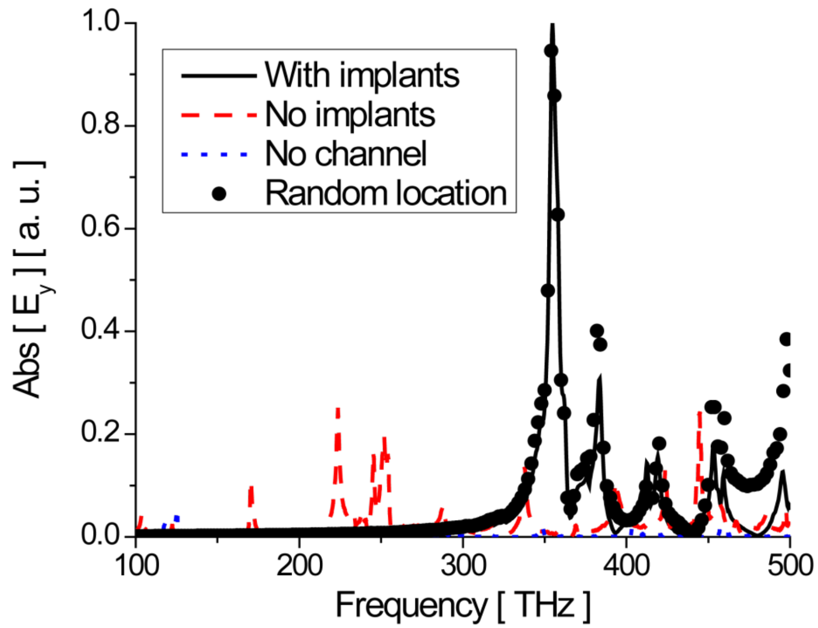
$a_{ch} = 20nm$

$t = 300nm$

A. Alù and N. Engheta
Phys. Rev. Lett. 2009

Enhancement of Optical Emitters

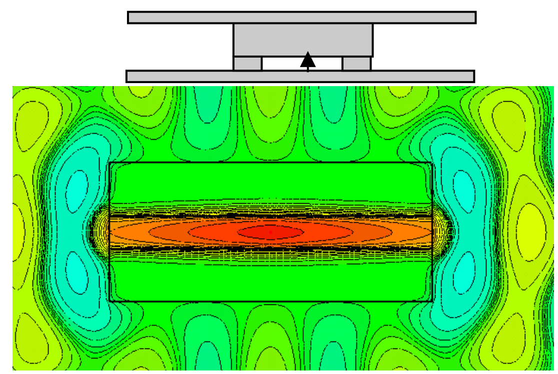




$$a = b = 200nm$$

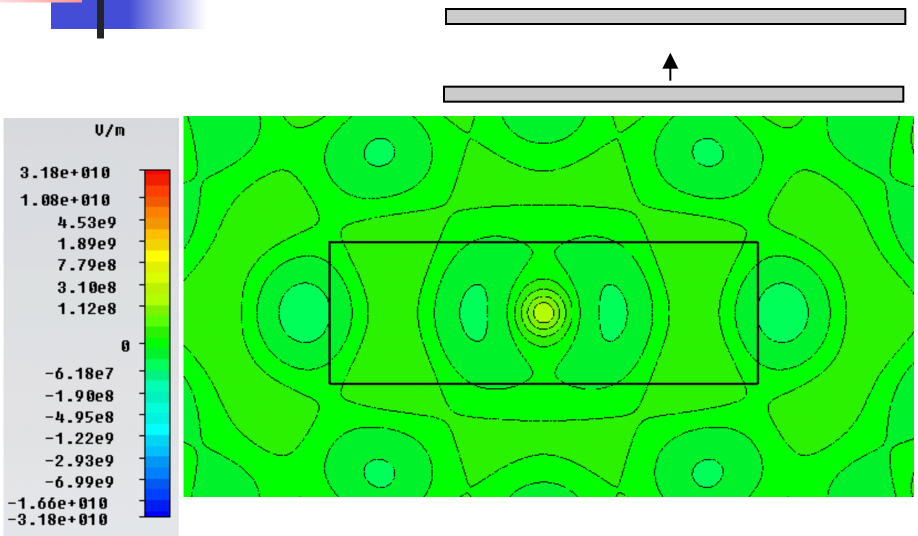
$$a_{ch} = 20nm$$

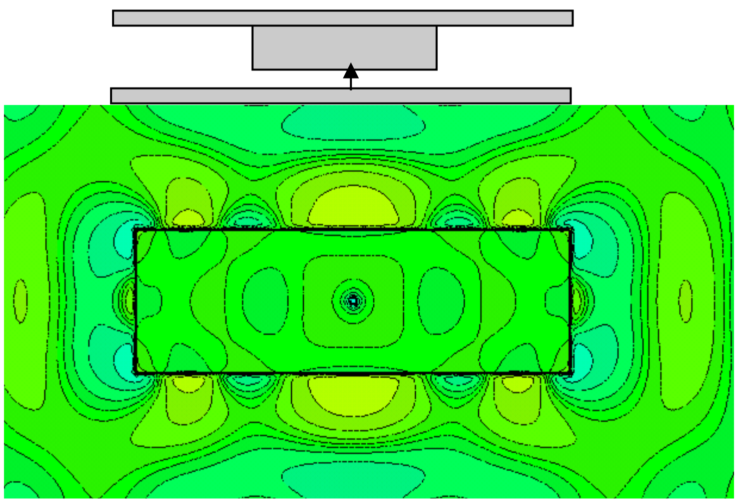
$$t = 50nm$$

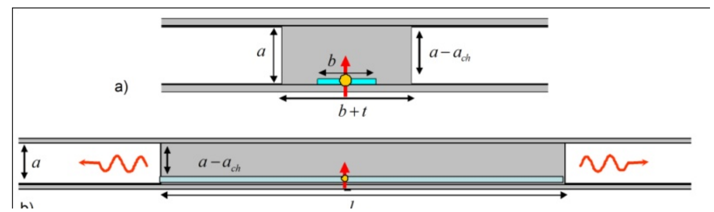


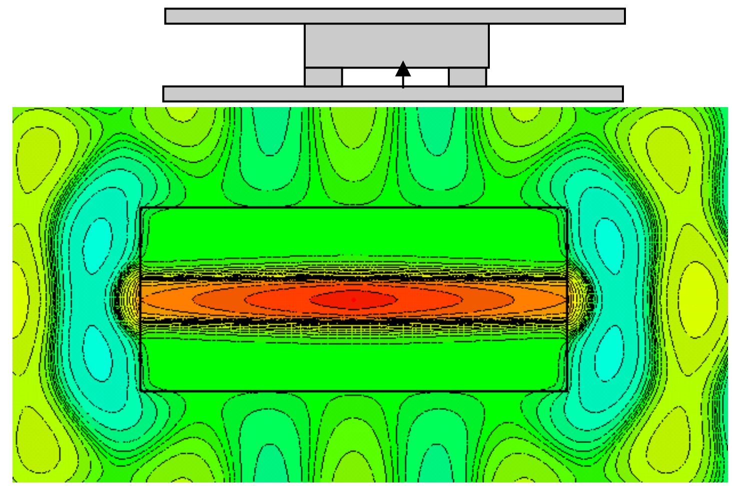
A. Alù and N. Engheta
Phys. Rev. Lett. 103, 043902 (2009)

ENZ and Purcell Effects



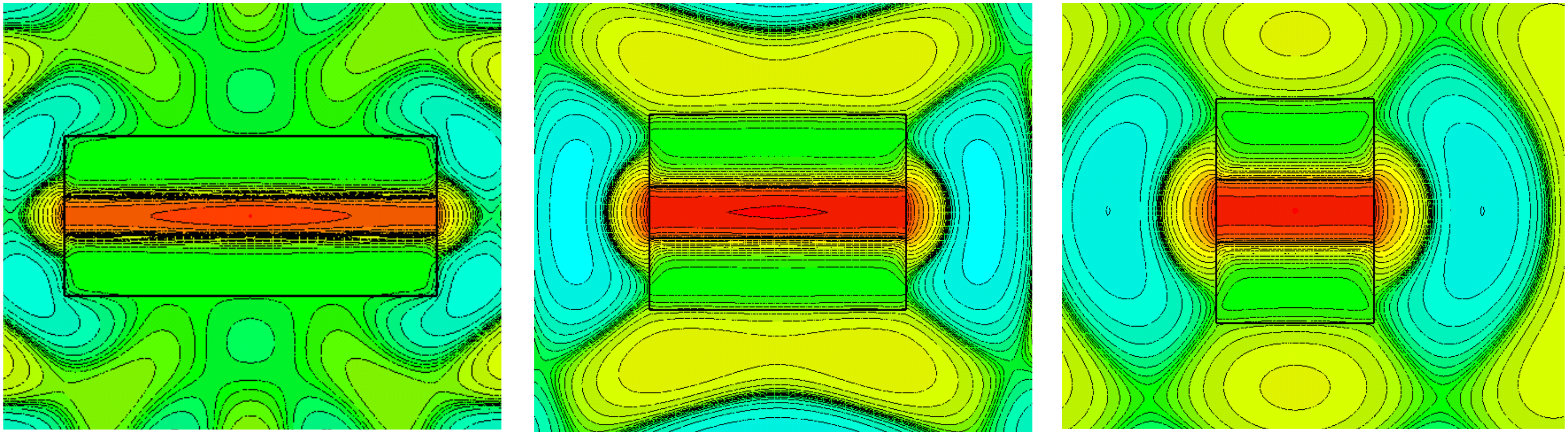






A. Alù and N. Engheta
Phys. Rev. Lett. 2009

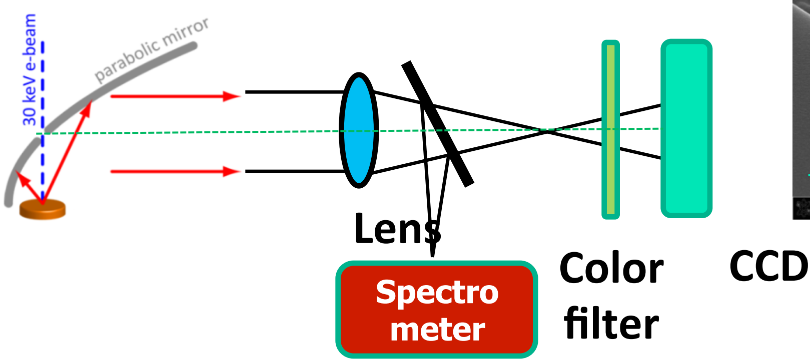
ENZ and Purcell Effects

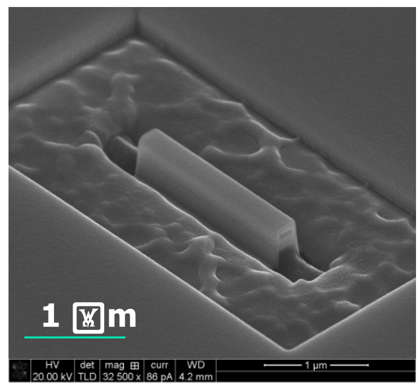


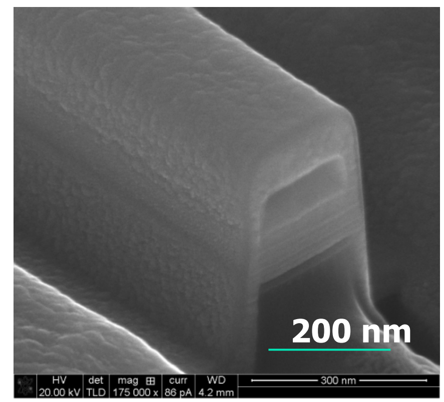
A. Alù and N. Engheta
Phys. Rev. Lett. 2009

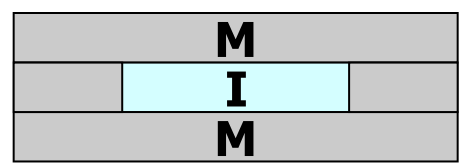
Experimental Verification Using CL Spectroscopy

Collaboration with Albert Polman's Group in AMOLF





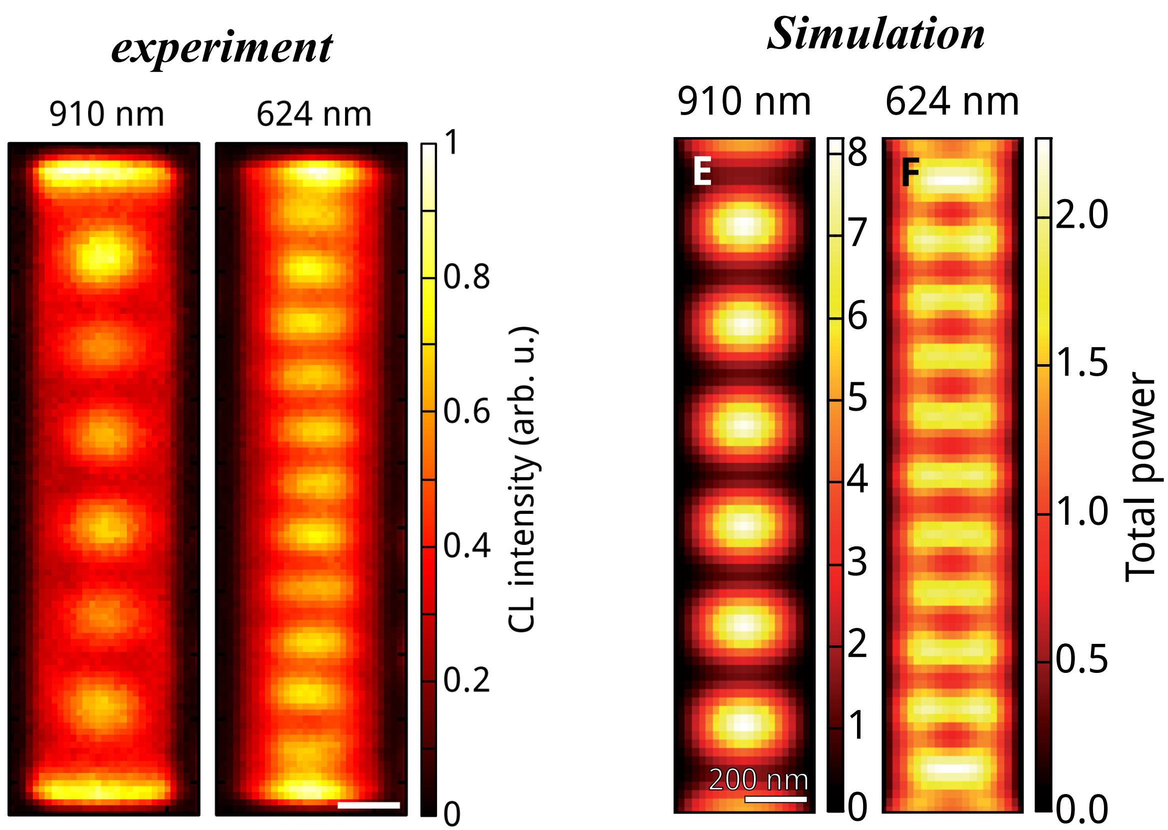




E. J. Vesseur, T. Coenen, H. Caglayan, N. Engheta, A. Polman *Phys. Rev. Lett., 110, 013902 (2013)*

Experimental Verification Using CL Spectroscopy

Collaboration with Albert Polman's Group in AMOLF



E. J. Vesseur, T. Coenen, H. Caglayan, N. Engheta, A. Polman *Phys. Rev. Lett., 110, 013902 (2013)*

Experimental Verification Using CL Spectroscopy

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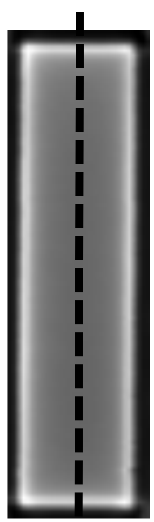
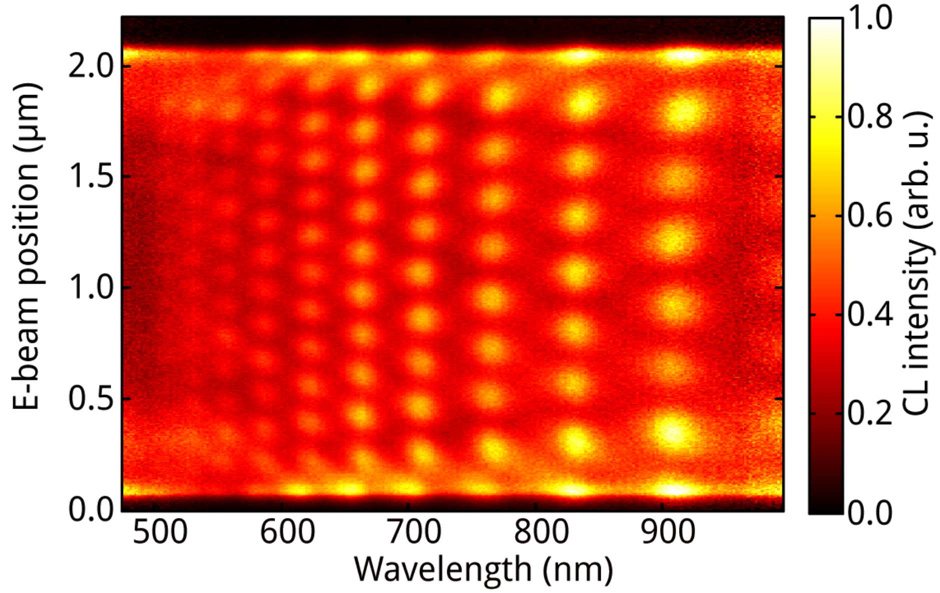


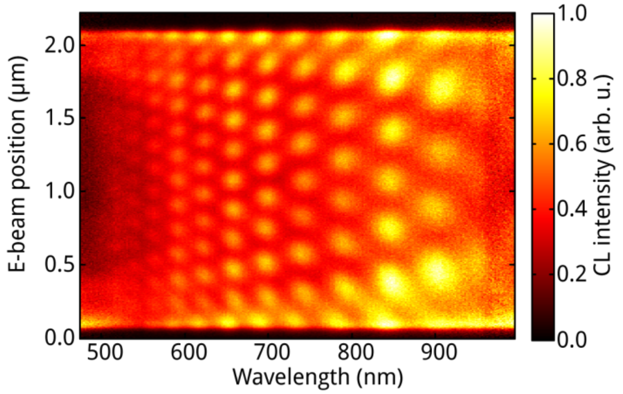


E. J. Vesseur, T. Coenen, H. Caglayan, N. Engheta, A. Polman Phys. Rev. Lett., (2013)

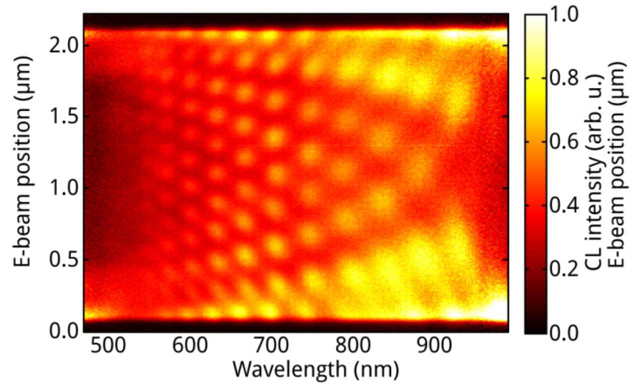
Experimental Verification Using CL Spectroscopy

Collaboration with Albert Polman's Group in AMOLF

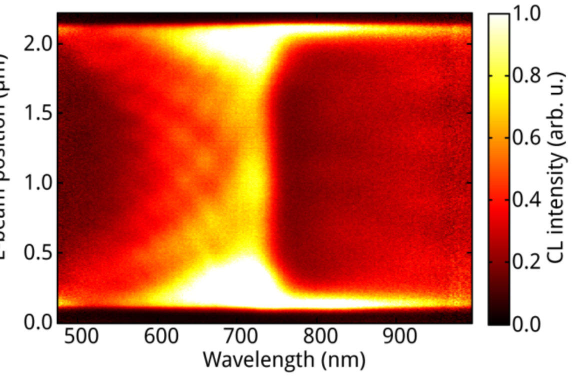
Waveguide Width = 250 nm



Waveguide Width = 240 nm



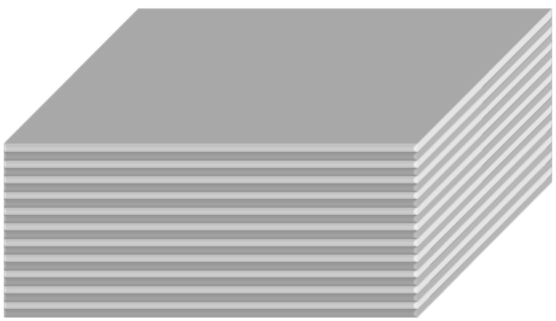
Waveguide Width = 180 nm



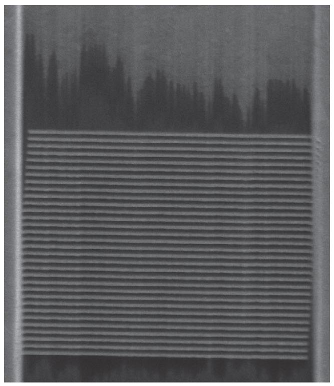
E. J. Vesseur, T. Coenen, H. Caglayan, N. Engheta, A. Polman *Phys. Rev. Lett., 110, 013902 (2013)*

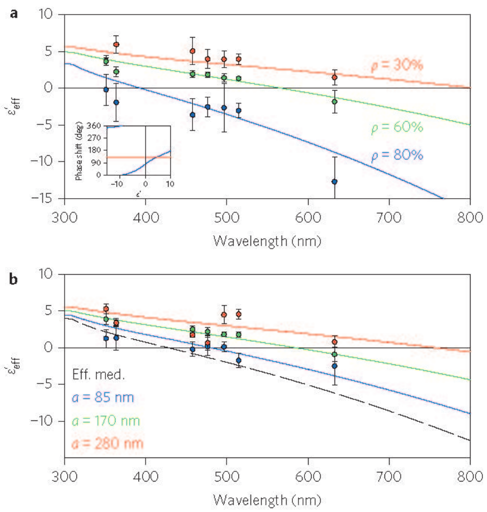
Experimental Verification ENZ Stack

Collaboration with Albert Polman's Group in AMOLF



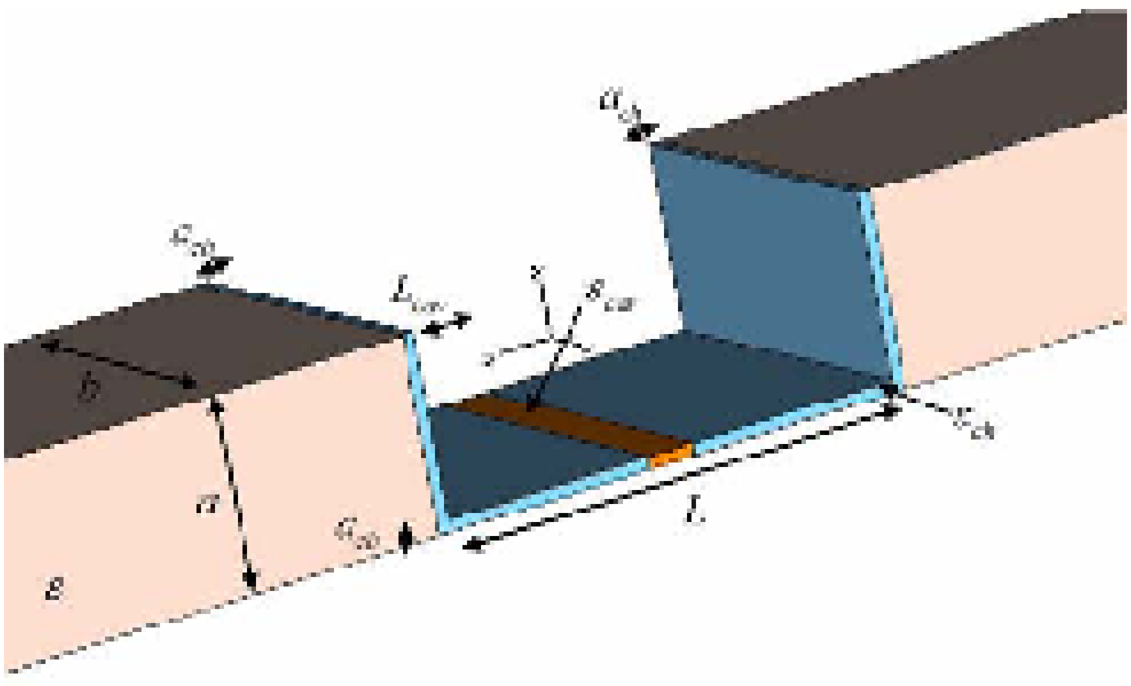
$\mathrm{Re}(\varepsilon) \cong 0$





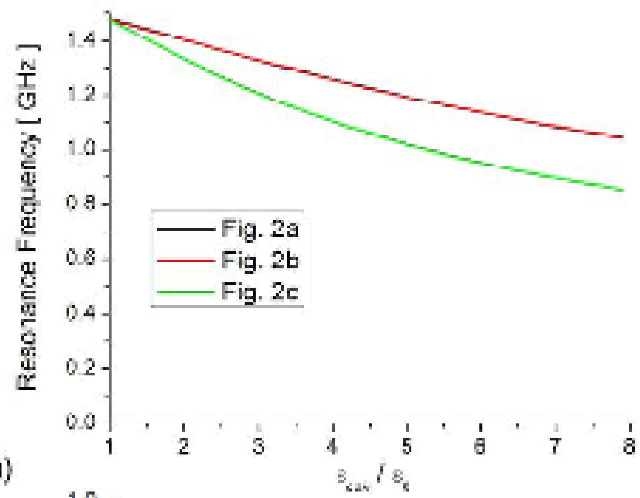
R. Maas, J. Parsons, N. Engheta, A. Polman Nature Photonics, 7(11), 907-912 (2013)

Dielectric Sensing

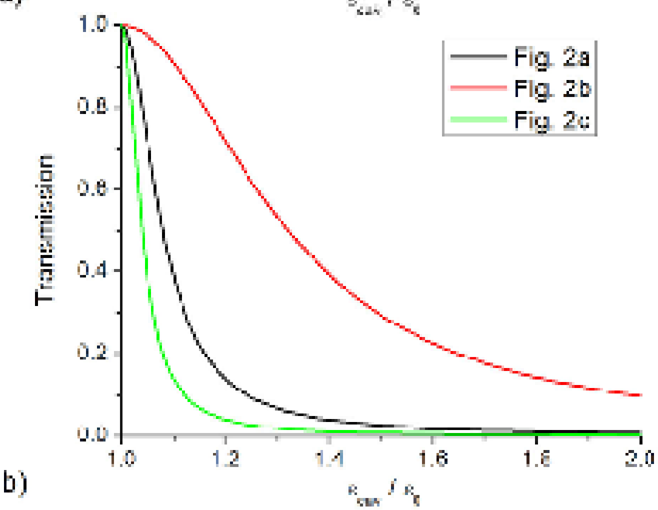


A. Alù and N. Engheta, Phys. Rev. B., 78, July 2008

Dielectric Sensing



a)

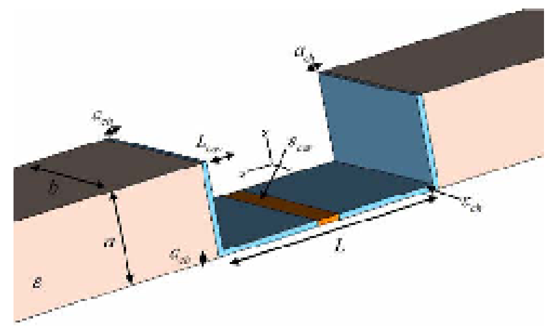


b)

$L_{cav} = L/10$
$a_{ch} = a/64$

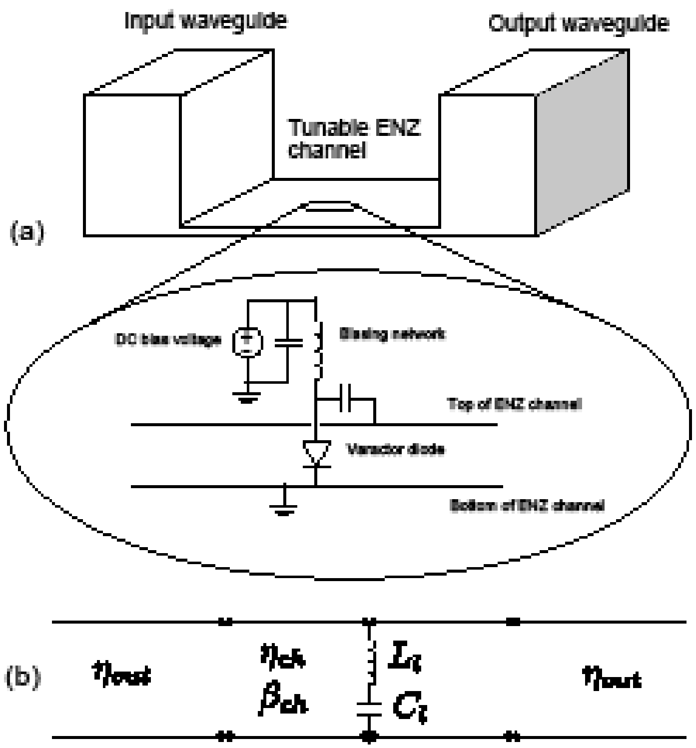
$L_{cav} = L/10$
$a_{ch} = a/16$

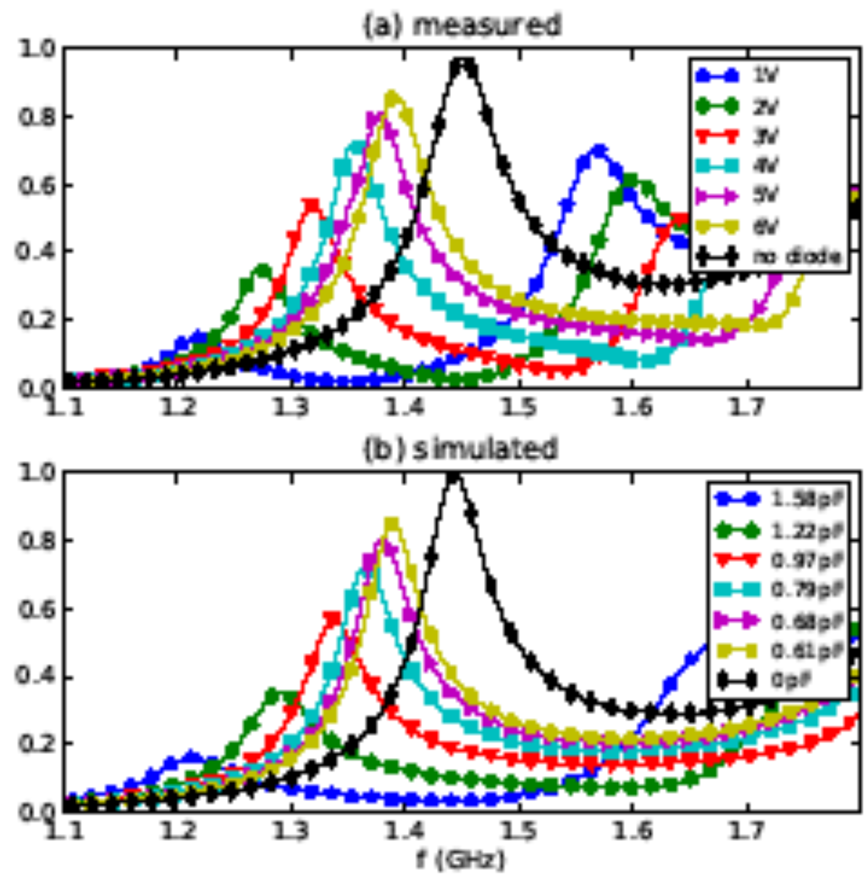
$L_{cav} = L/5$
$a_{ch} = a/64$



A. Alù and N. Engheta, Phys. Rev. B., 78, July 2008

Nonlinearity in ENZ Channels





D. Powell, A. Alù, B. Edwards, A. Vakil, Y. Kivshar, and N. Engheta,
Phys. Rev. B. 2009.

Fields and Waves in Metamaterials Part 3

Nader Engheta

University of Pennsylvania
Philadelphia, PA 19104, USA

August 16-17, 2014

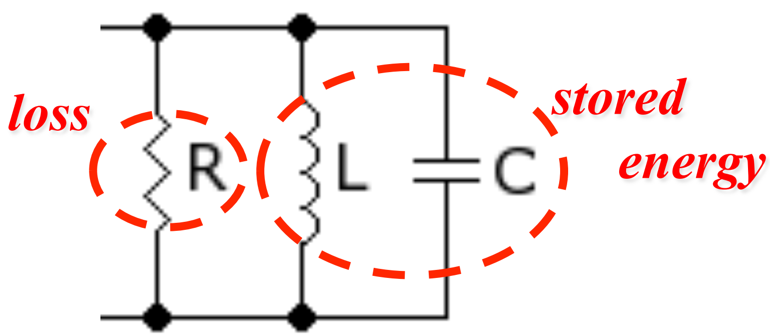
Graphene

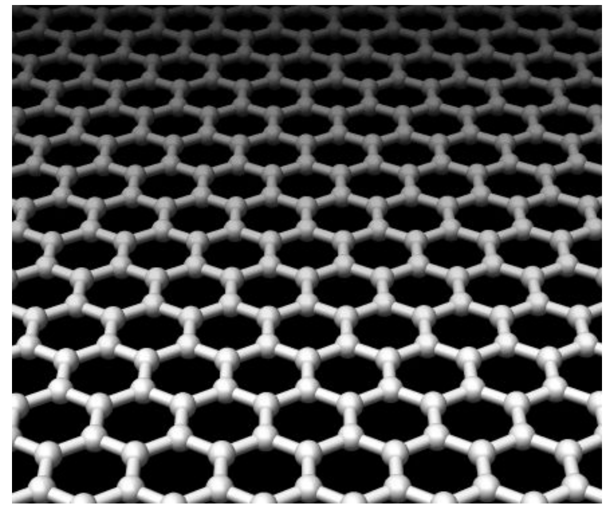
$$J_s = \sigma_g E$$

$$I = \sigma_g V = Y \cdot V$$

$$\sigma_g = \underset{>0}{\sigma_{g,r}} + i\underset{>0 \text{ or } <0}{\sigma_{g,i}}$$

$$Y = G + i\,B$$





http://math.ucr.edu/home/baez/graphene.jpg

Graphene Conductivity

$$\sigma_g(\omega,\mu_c,\Gamma,T)=$$
$$=\frac{-ie^2(\omega+i2\Gamma)}{\pi\hbar^2}\left[\frac{1}{(\omega+i2\Gamma)^2}\int_0^\infty \Omega\left(\frac{\partial f_d(\Omega)}{\partial\Omega}-\frac{\partial f_d(-\Omega)}{\partial\Omega}\right)d\Omega-\int_0^\infty\frac{f_d(-\Omega)-f_d(\Omega)}{(\omega+i2\Gamma)^2-4(\Omega/\hbar)^2}\Omega\right]$$

$$\sigma_g=\sigma_{\text{interband}}+\sigma_{\text{intraband}}$$

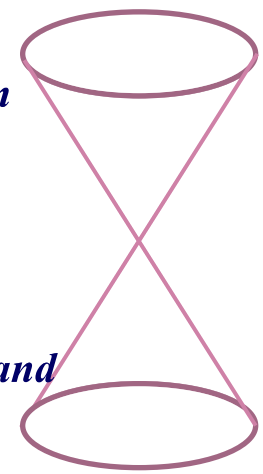
$$f_d(\Omega)\equiv\left[e^{(\Omega-\mu_c)/k_BT}+1\right]^{-1}$$

$$\sigma_{\text{interband}}\approx\frac{ie^2}{4\pi\hbar}\ln\left[\frac{2|\mu_c|-(\omega+i2\Gamma)\hbar}{2|\mu_c|+(\omega+i2\Gamma)\hbar}\right]$$

$$k_BT<<|\mu_c|$$

$$\text{Im}(\sigma_{\text{interband}})<0$$

Conduction Band



Valence Band

$$\sigma_{\text{intraband}}=\frac{ie^2k_BT}{\pi\hbar^2(\omega+i2\Gamma)}\left[\frac{\mu_c}{k_BT}+2\ln\left(e^{-\mu_c/k_BT}+1\right)\right]$$

$$\text{Im}(\sigma_{\text{intraband}})>0$$

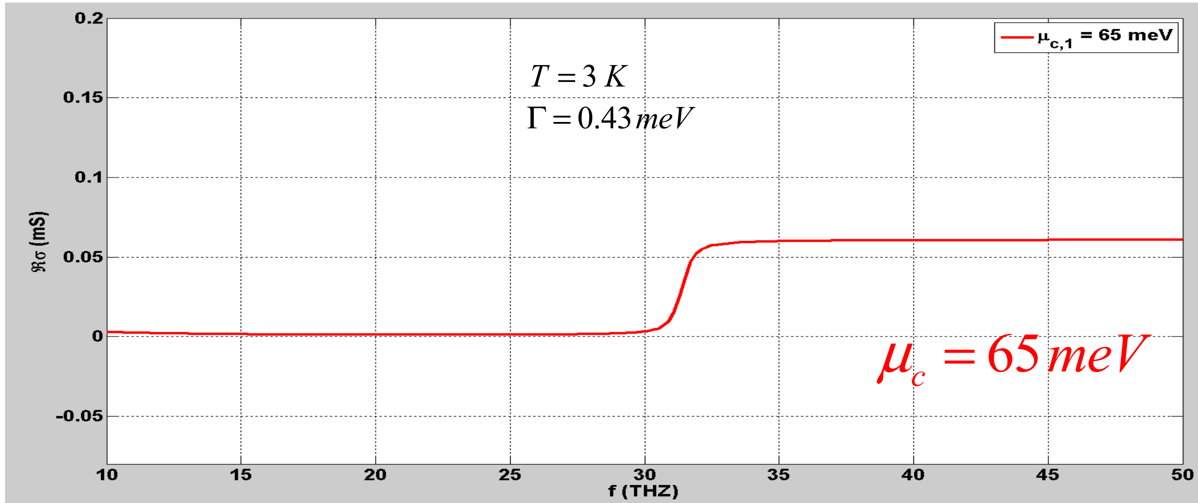
P. Gusynin et al., J. Phys: Condens. Matter, 19 (2007)

G. Hanson, J. Appl. Phys. 103, 064302 (2008)

Graphene Conductivity

$$\sigma_g = \sigma_{\text{interband}} + \sigma_{\text{intraband}}$$

$\text{Re}(\sigma_g)$



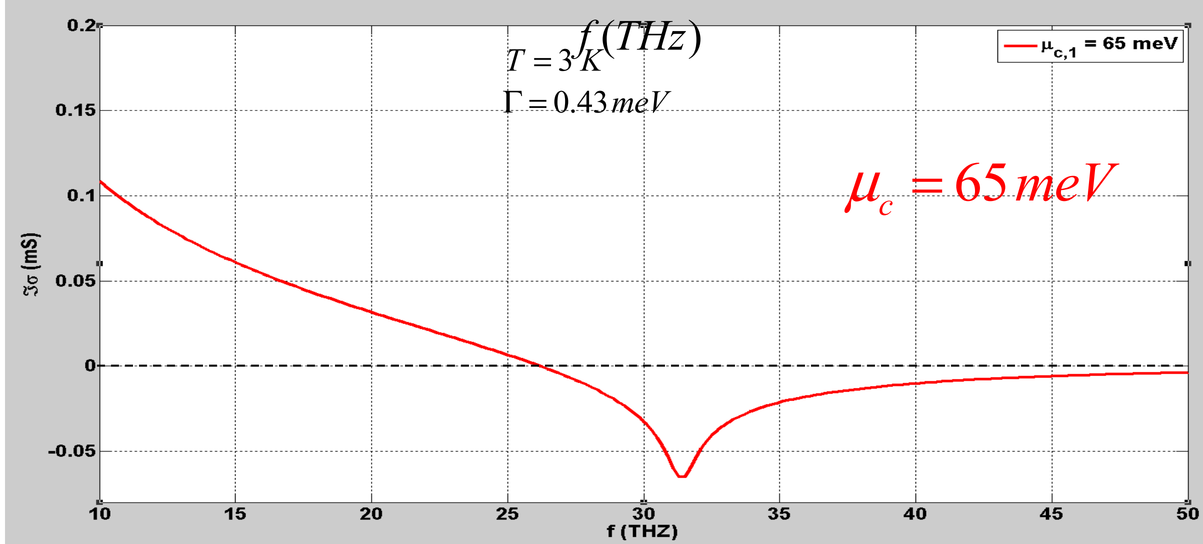
$f\,(THz)$

$$\sigma_g = \sigma_{g,r} + i\sigma_{g,i}$$

$$\sigma_{g,r} = f_1(\omega, \mu_c, \Gamma, T)$$

$$\sigma_{g,i} = f_2(\omega, \mu_c, \Gamma, T)$$

$\text{Im}(\sigma_g)$



$f\,(THz)$

P. Gusynin et al., J. Phys: Condens. Matter, 19 (2007)

G. Hanson, J. Appl. Phys. 103, 064302 (2008)

Graphene Conductivity

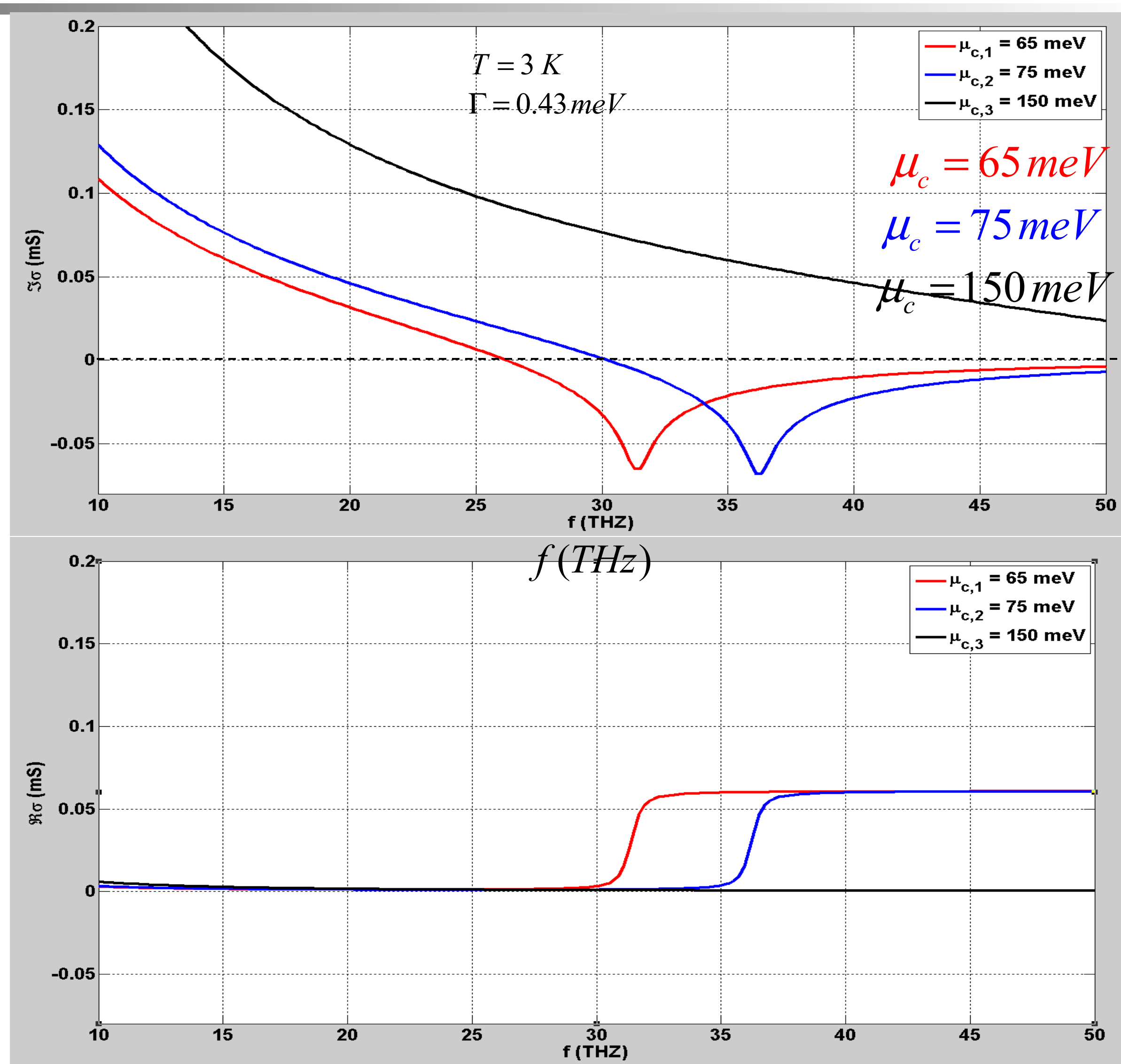
$$\sigma_g = \sigma_{g,r} + i\sigma_{g,i}$$

$$\sigma_{g,r} = f_1(\omega, \mu_c, \Gamma, T)$$

$$\sigma_{g,i} = f_2(\omega, \mu_c, \Gamma, T)$$

$\mathrm{Im}(\sigma_g)$

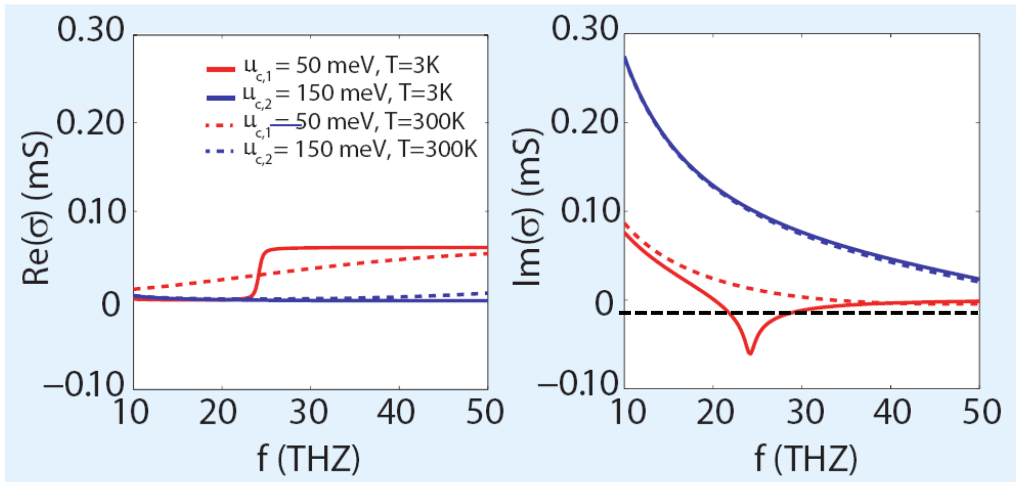
$\mathrm{Re}(\sigma_g)$

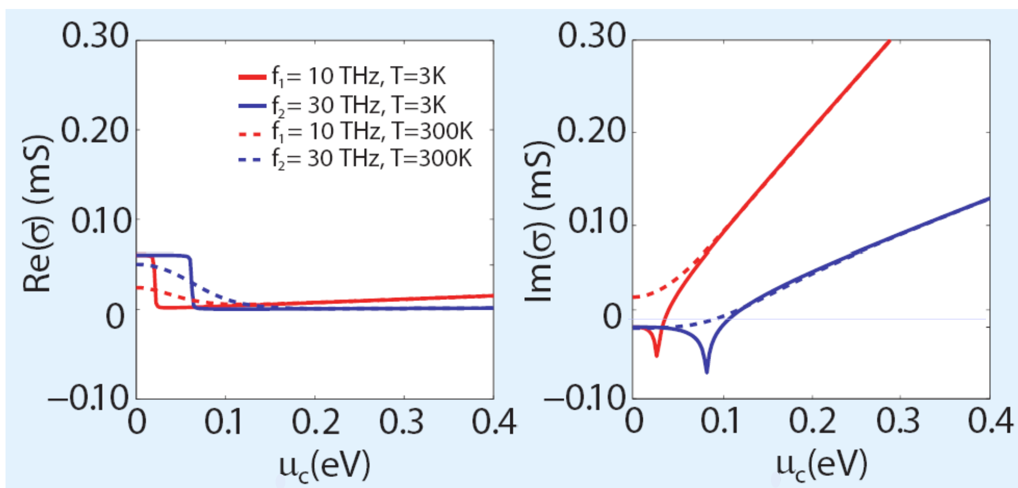


G. Hanson, J. Appl. Phys. 103, 064302 (2008)

$f\,(THz)$

Graphene Conductivity





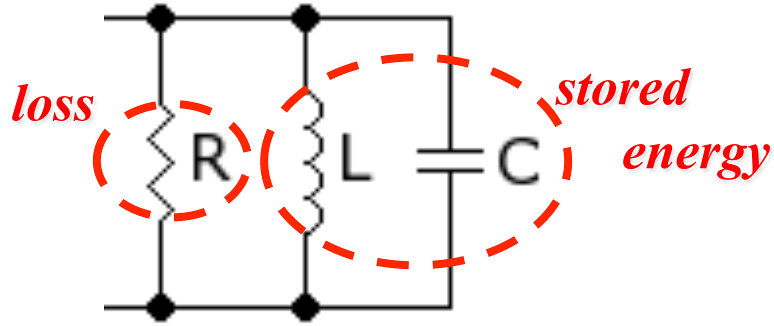
$\Gamma = 0.43$ meV

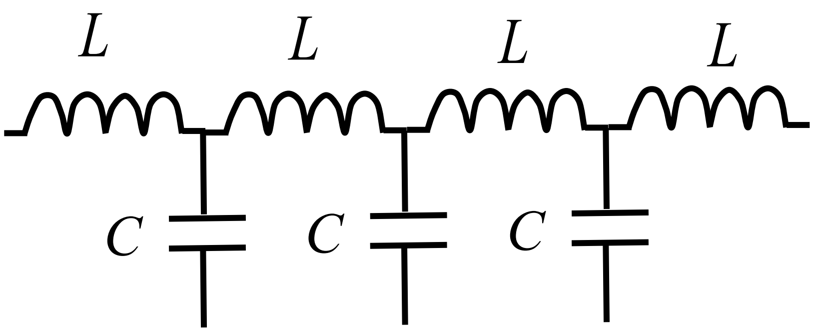
$$\sigma_g = \sigma_r + i\sigma_i$$

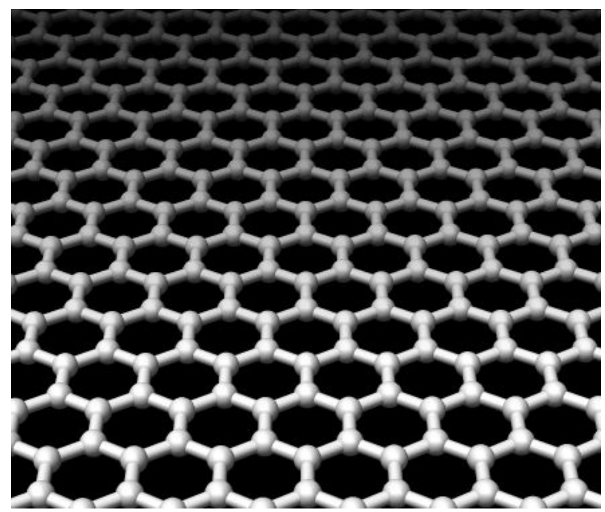
G. Hansen, J. Appl. Phys. 103, 064302 (2008)

From Transmission Line to Graphene

$$\sigma_g = \sigma_{g,r} + i\sigma_{g,i}$$

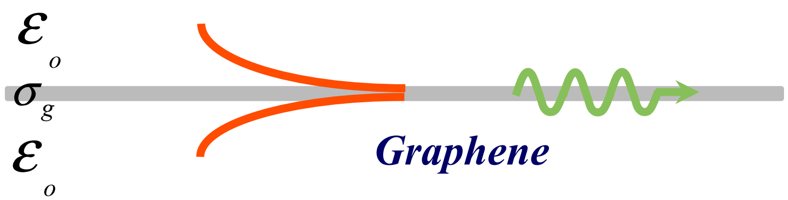






http://math.ucr.edu/home/baez/graphene.jpg

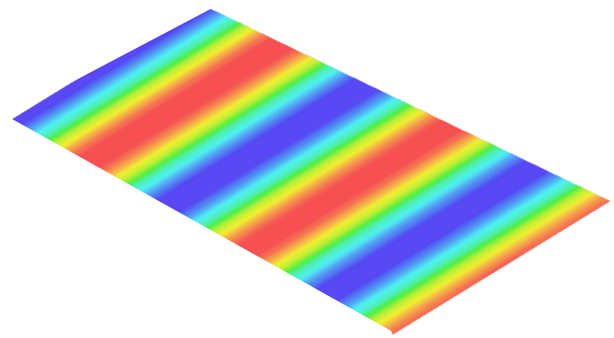
SPP along Graphene



$$\beta_{SPP} = \omega\sqrt{\varepsilon_o\mu_o}\sqrt{1-\left(\frac{2}{\sigma_g\sqrt{\mu_o/\varepsilon_o}}\right)^2} \qquad \sigma_{g,i} > 0$$

$$\beta_{SPP} >> \omega\sqrt{\varepsilon_o\mu_o}$$

$$\lambda_{SPP} << \lambda_{free-space} \qquad \beta_{SPP} = n_{SPP}k_o$$



$$\lambda_{SPP} \approx \frac{\lambda_o}{70} \approx 144nm \qquad \beta_{SPP} \approx 70k_o$$

S. A. Mikhailov, K. Ziegler, Phys. Rev. Lett. 99, 016803 (2007)

G. Hanson, J. Appl. Phys. 103, 064302 (2008)

M. Jablan, H. Buljan, M. Soljacic, Phys. Rev. B., 80, 245435 (2010)

Tailoring Conductivity and SPP

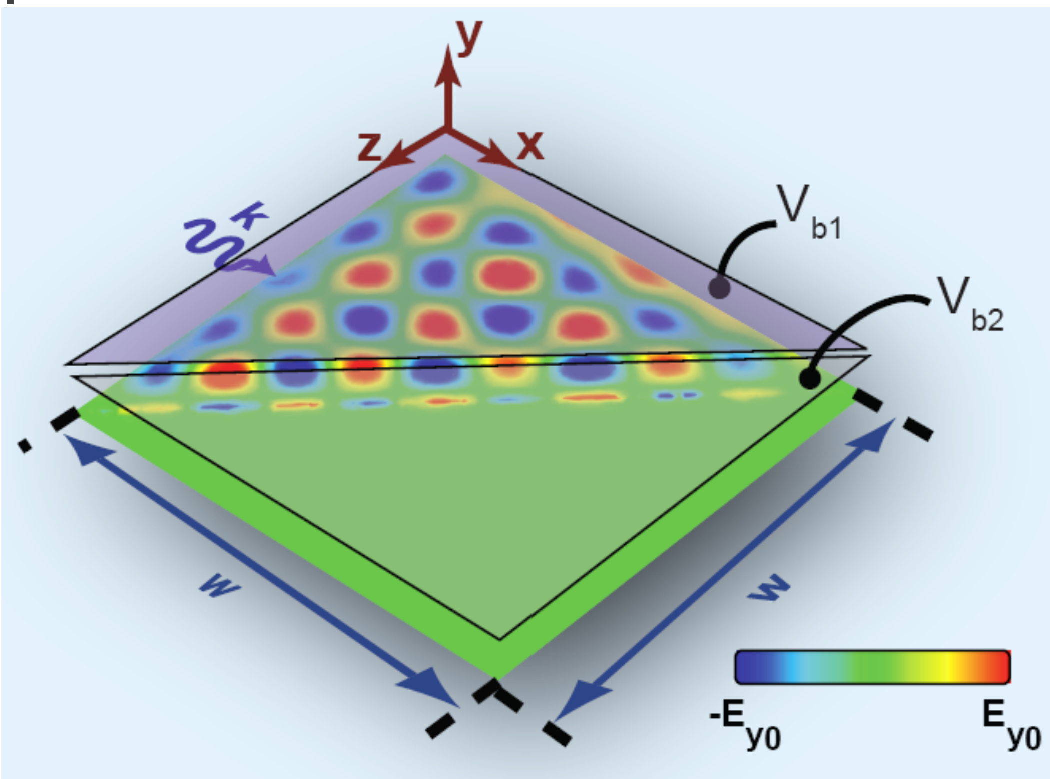
$$\sigma_{g,i} = f(\omega, \mu_c, \Gamma, T)$$

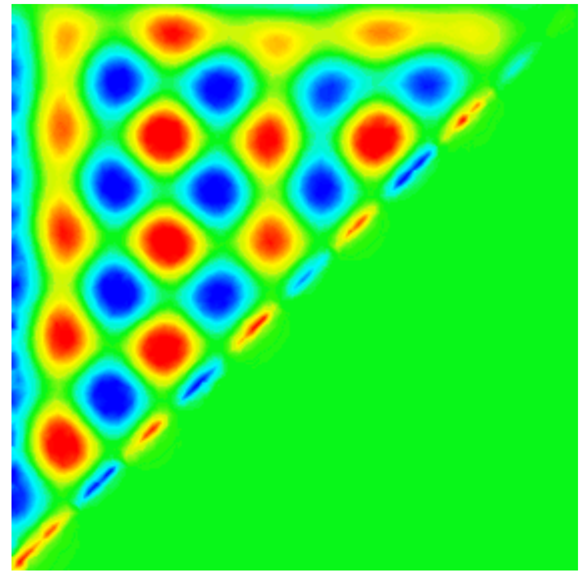
cnst cnst cnst

$$n_{SPP} = \frac{\beta_{SPP}}{k_0} \propto \frac{1}{\sigma_{g,i}}$$

$\sigma_{g,i}$? $\sigma_{g,r}$

Fresnel Reflection





w= 800 nm

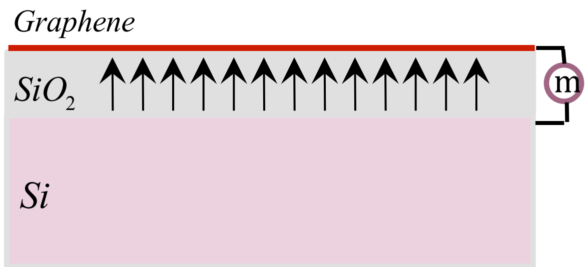
T = 3 K, Γ = 0.43 meV

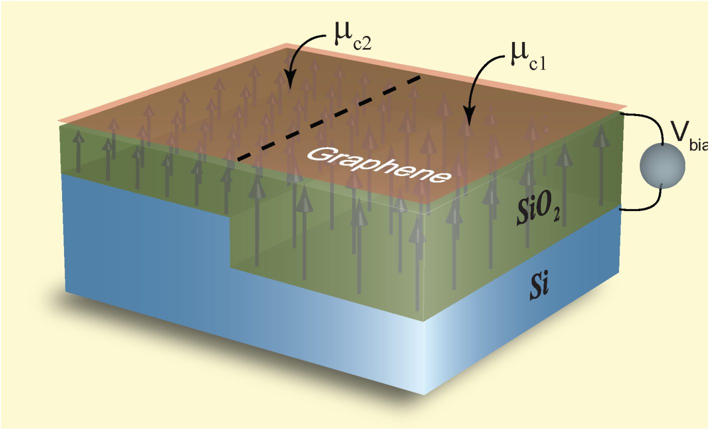
$m_{c,1}$ = 150 meV → σ_{g1} = 0.0009 + i 0.0765 mS

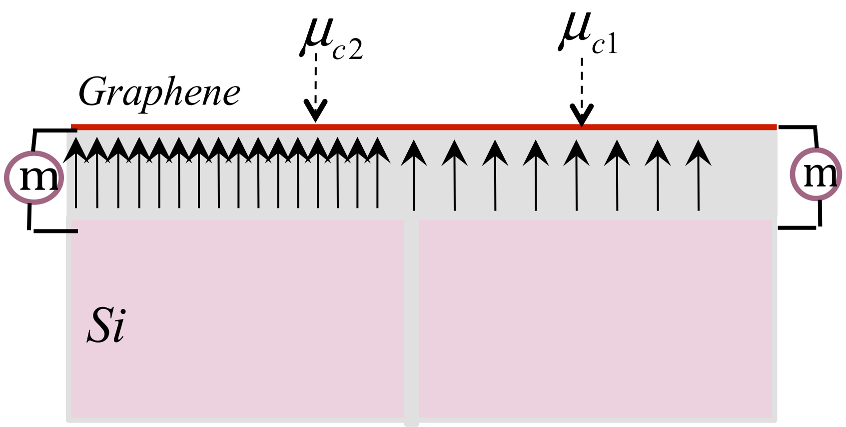
$m_{c,2}$ = 6.5 meV → σ_{g2} = 0.0039 − i 0.0324 mS

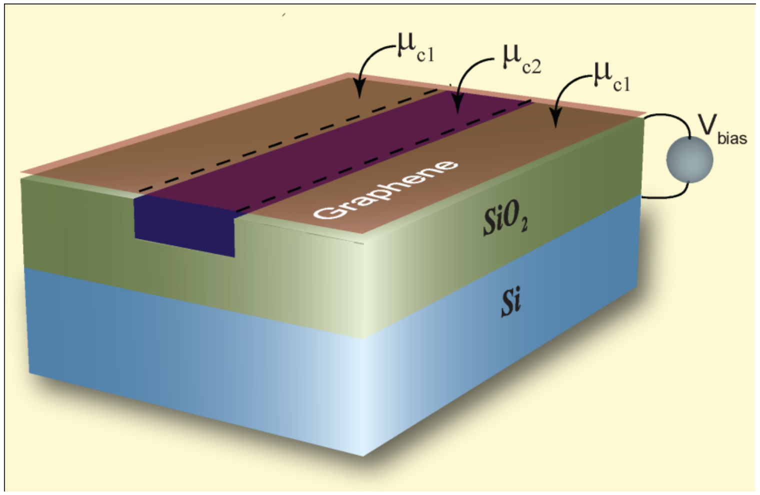
Vakil, Engheta, __Science__ 332, 1291 (2011)

Inhomogeneous Conductivity across Graphene



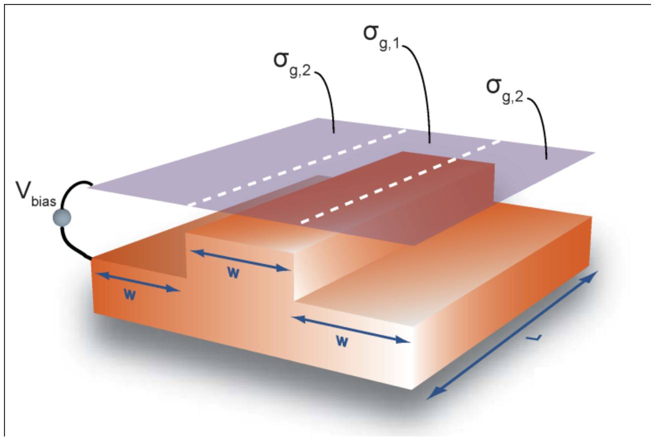


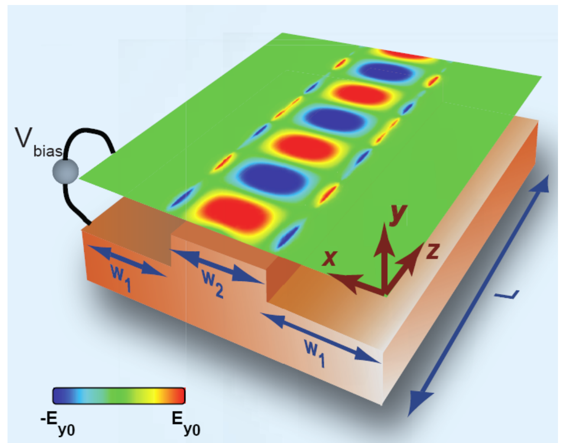


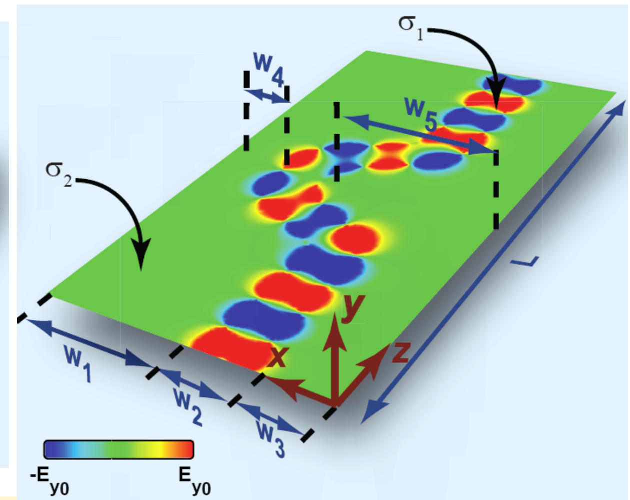


Vakil, Engheta, Science 332, 1291 (2011)

One-Atom-Thick Waveguides





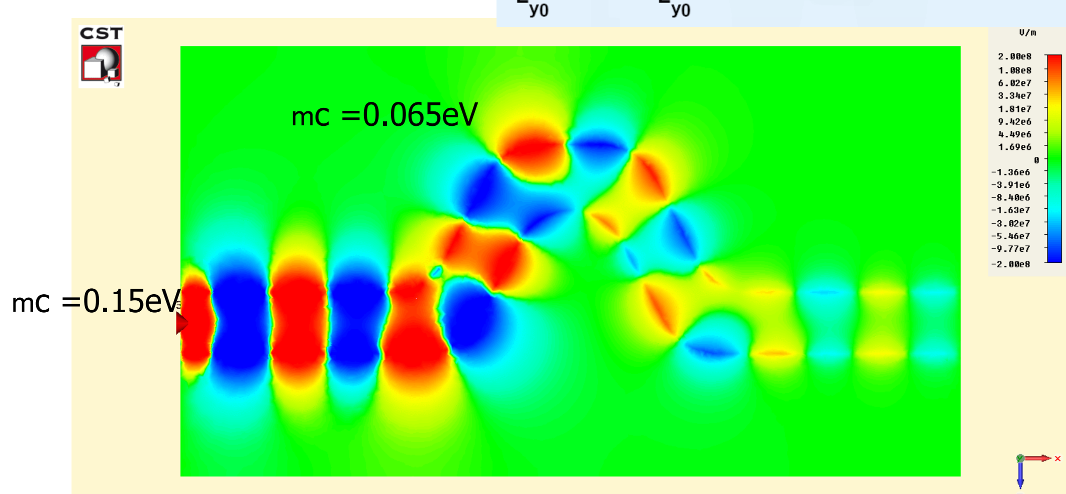


Region 1: $\sigma_{g,i} > 0$

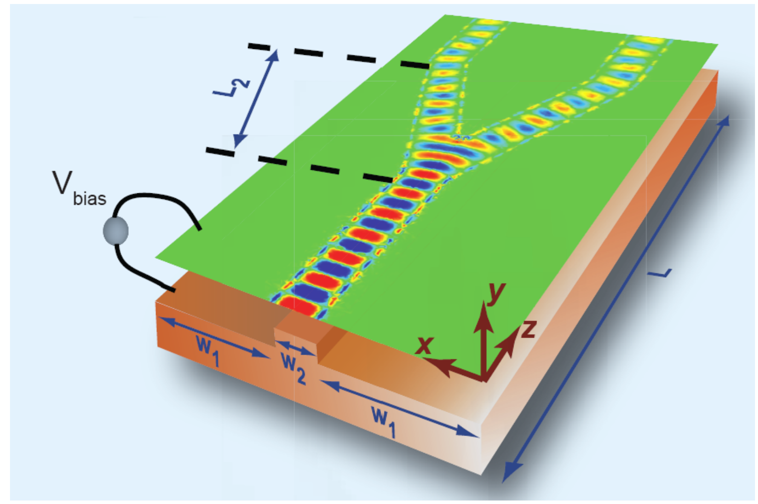
$\mu_c = 150\, meV$

Region 2: $\sigma_{g,i} < 0$

$\mu_c = 65\, meV$



One-Atom-Thick IR Splitter



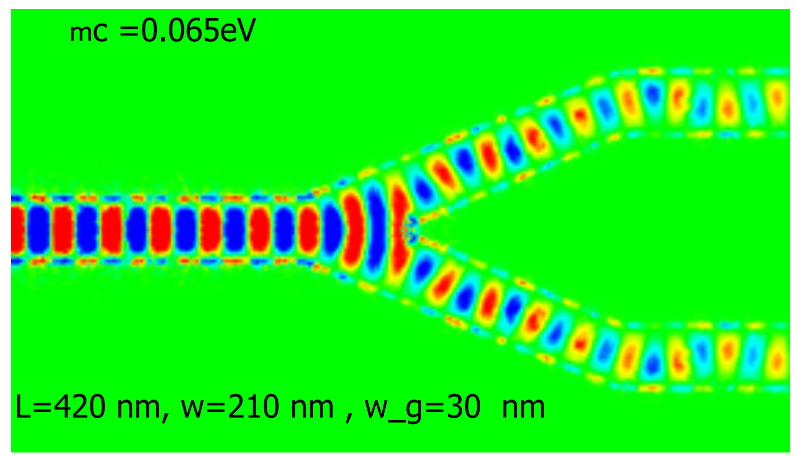
Region 1: $\sigma_{g,i} > 0$

$$\mu_c = 0.15\, eV$$

Region 2: $\sigma_{g,i} < 0$

$$\mu_c = 0.065\, eV$$

mc =0.15eV



One-Atom-Thick Optical "Fiber"

Region 1: $\sigma_{g,i} > 0$

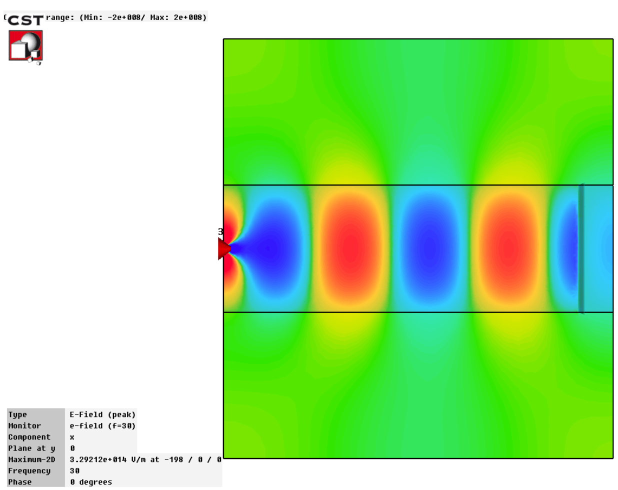
$\mu_c = 150\ meV$

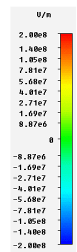
$\mathrm{Re}(\beta_{SPP}) \approx 70k_o$

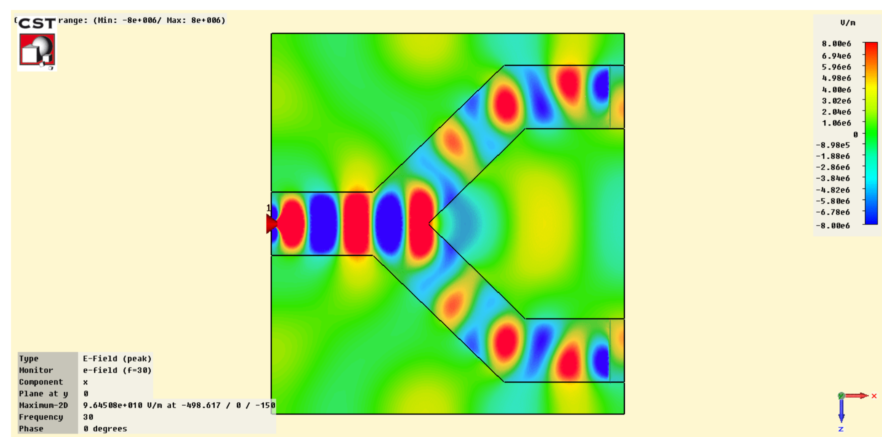
Region 2: $\sigma_{g,i} > 0$

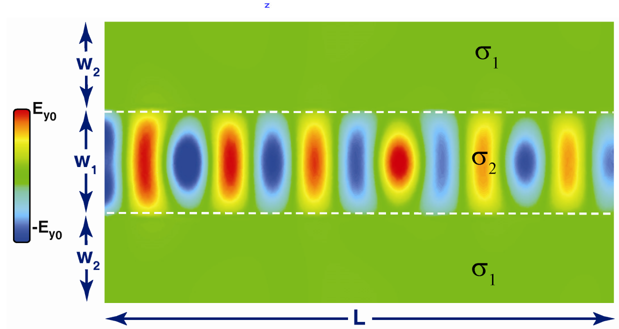
$\mu_c = 300\ meV$

$\mathrm{Re}(\beta_{SPP}) \approx 30k_o$

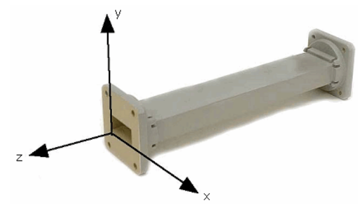
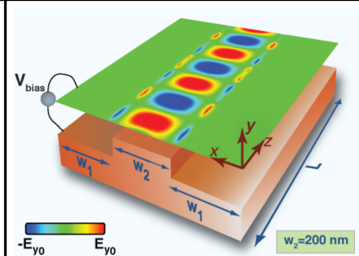
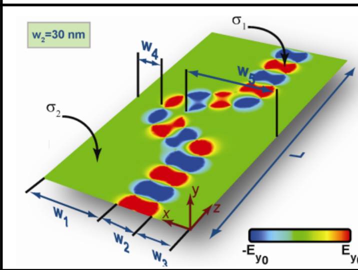
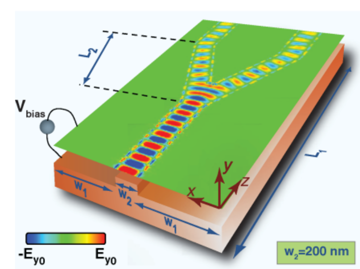
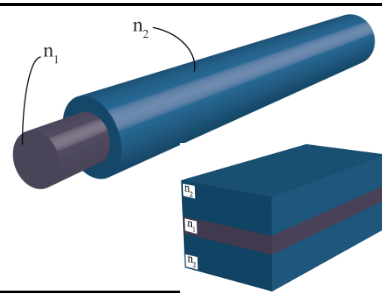
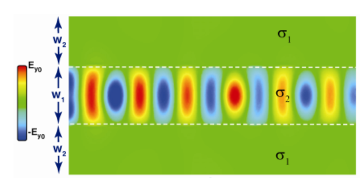








Guiding Waves on one-atom-thick Platform

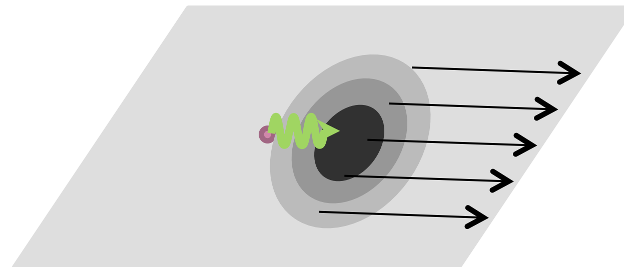
	3D component	One-Atom-Thick Version
Waveguide		
Bent Waveguide		
Splitter/Divider		
Optical Fiber/ Dielectric Slab waveguide		

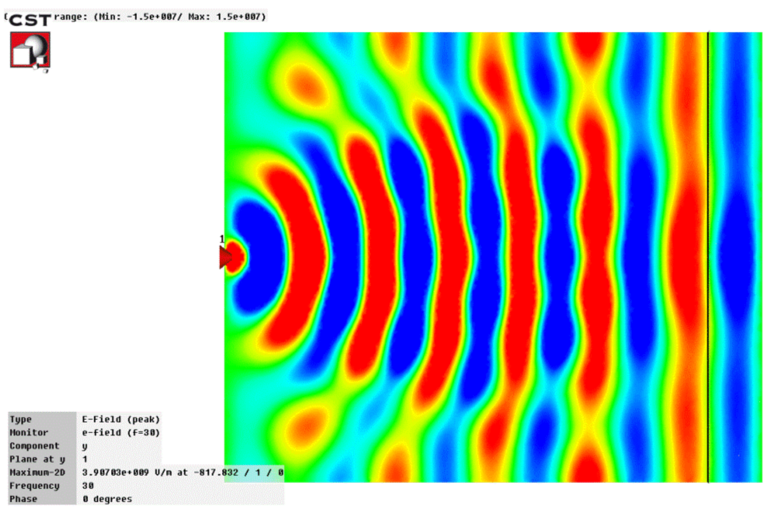
Graphene SPP Lens

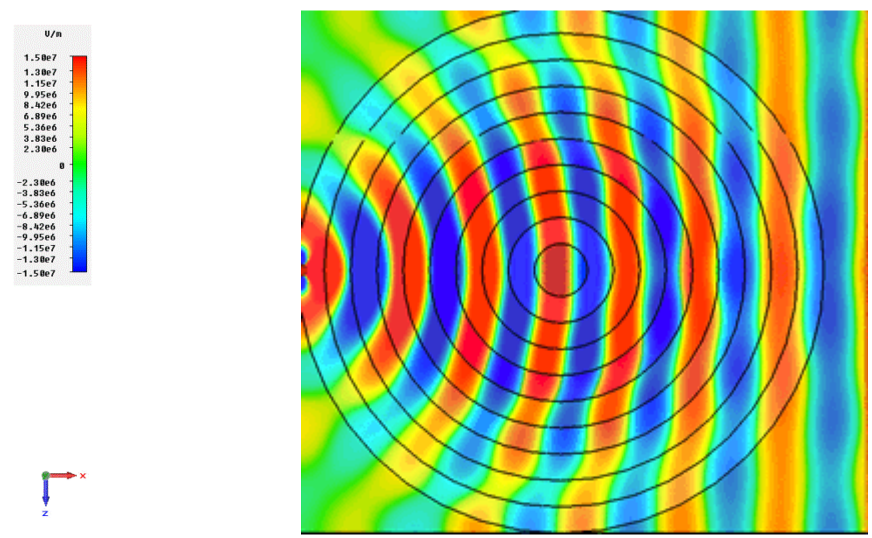
One-Atom-Thick TO: Lens

$$\beta_{SPP} \equiv n_{SPP} k_o >> k_o$$

$$\lambda_{SPP} << \lambda_{free-space}$$







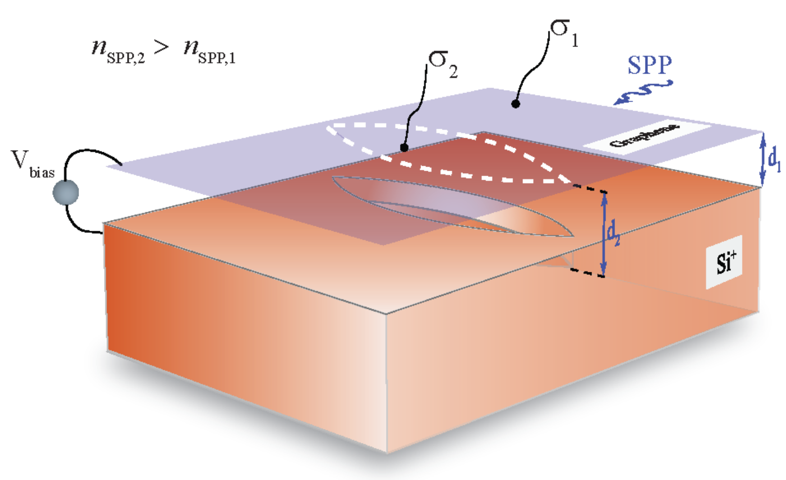
$D = 1.5\,\mu m$

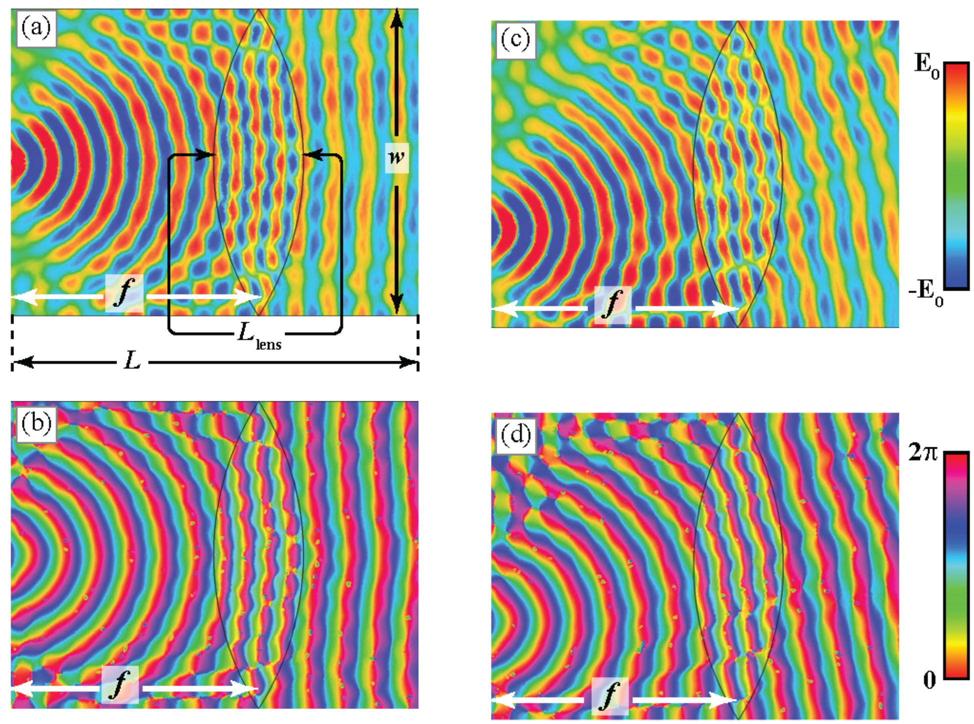
$L = 1.6\,\mu m$

$W = 75\ nm$

Vakil, Engheta, Science 332, 1291 (2011)

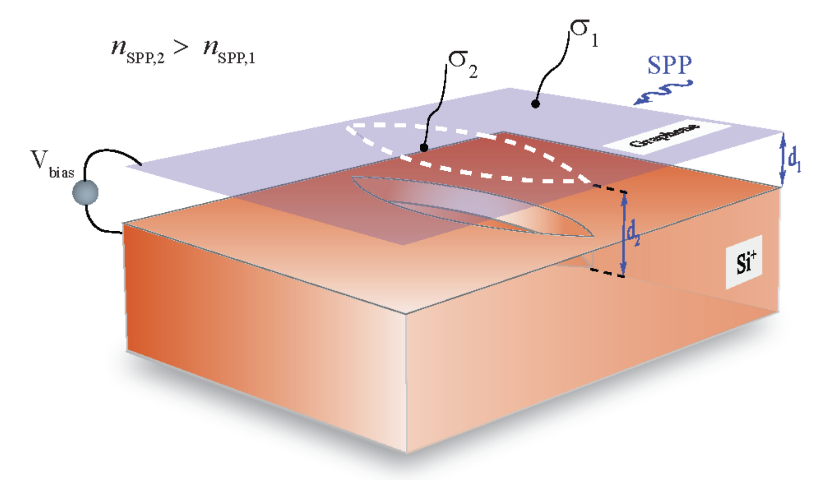
One-Atom-Thick Signal Processing: Fourier Transform

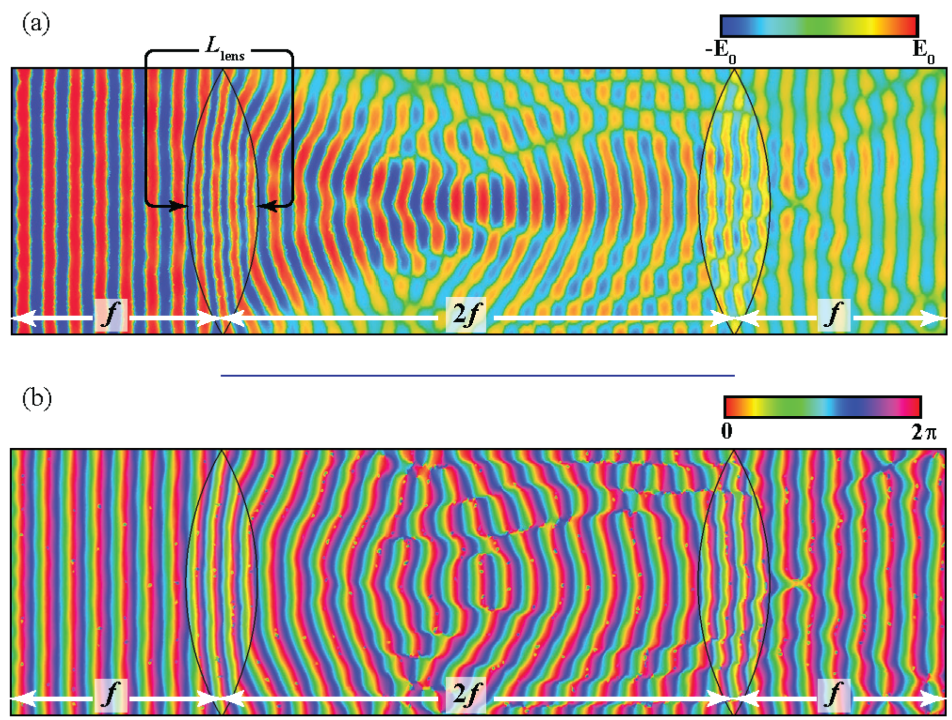




Vakil, Engheta, Phys. Rev. B, (2012)

Graphene Fourier Optics

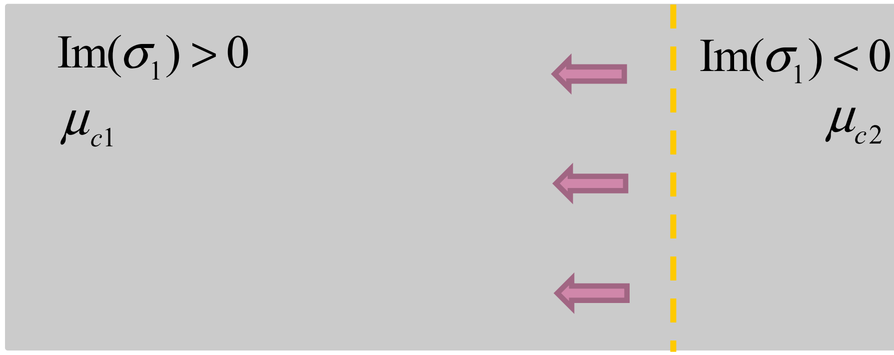


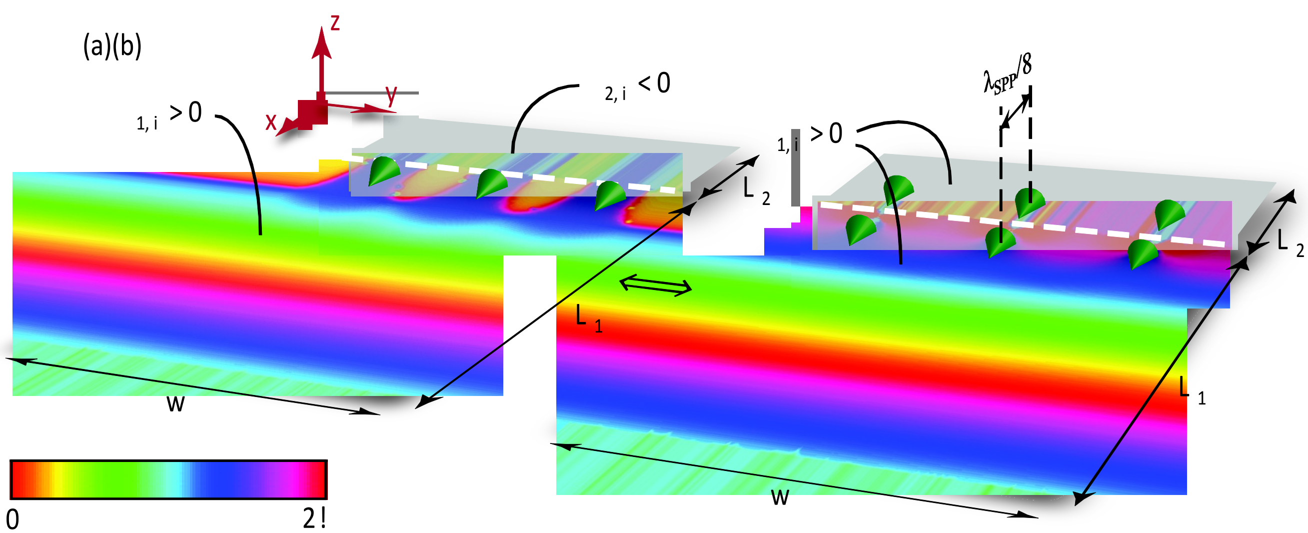


Vakil and Engheta, Phys. Rev. B (2012)

Graphene SPP Mirror

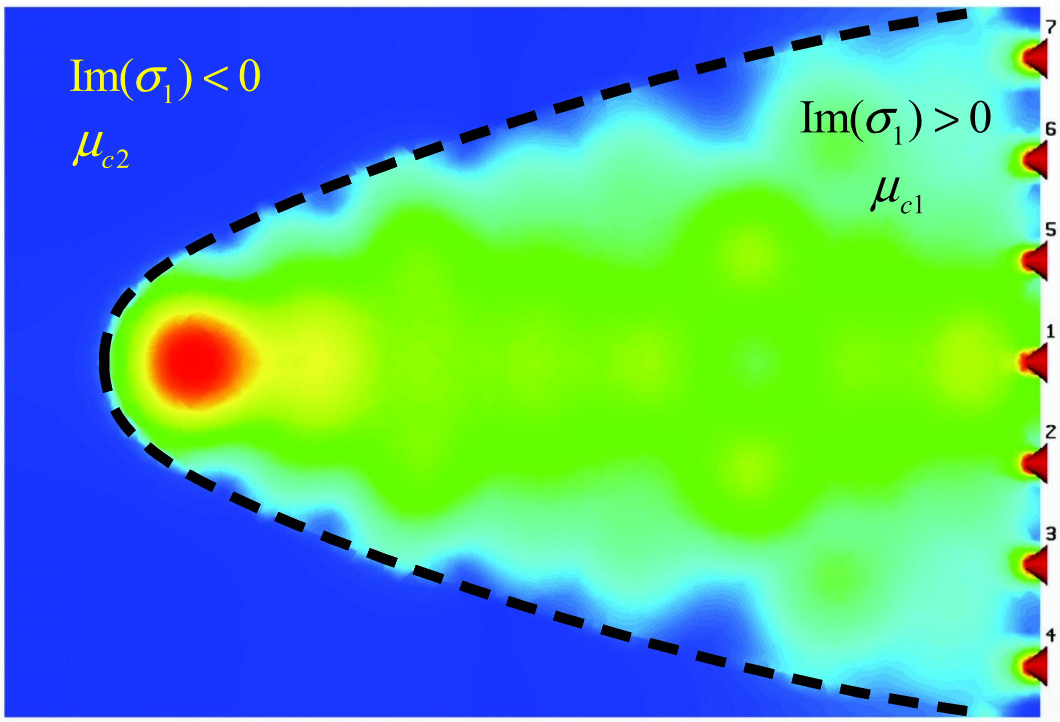
One-Atom-Thick SPP Reflector





Vakil and Engheta, Optics Communications, (2012)

One-Atom-Thick SPP Reflector



Vakil and Engheta, Optics Communications (2012)

Graphene Metamaterials

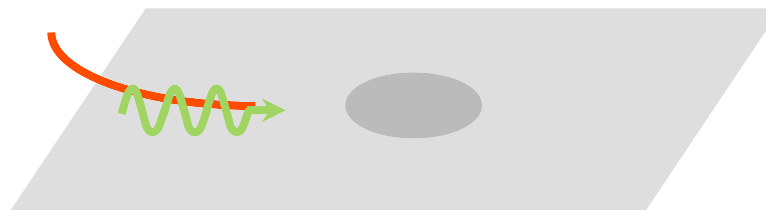
One-Atom-Thick Scatterer

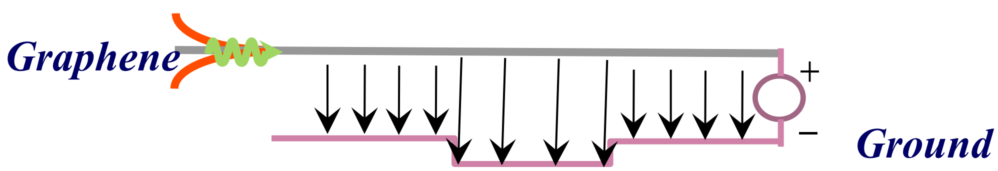
Region 1: $\sigma_{g,i} > 0$

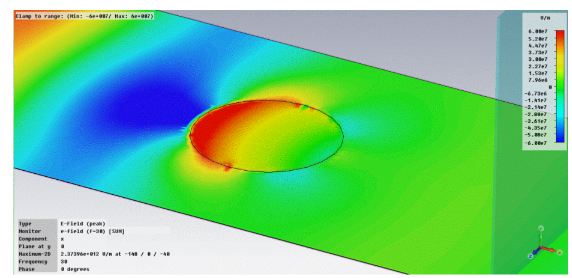
$$\mu_c = 150\, meV$$

Region 2: $\sigma_{g,i} < 0$

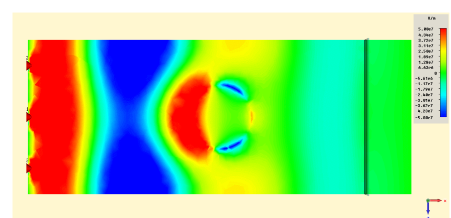
$$\mu_c = 65\, meV$$





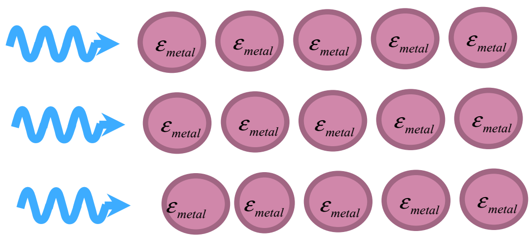


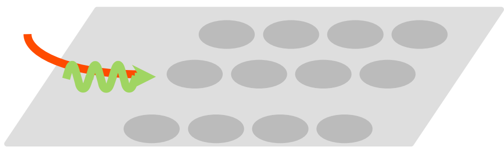
w=120nm
r=25nm



w=55 nm
D = 30 nm

One-Atom-Thick Metamaterials



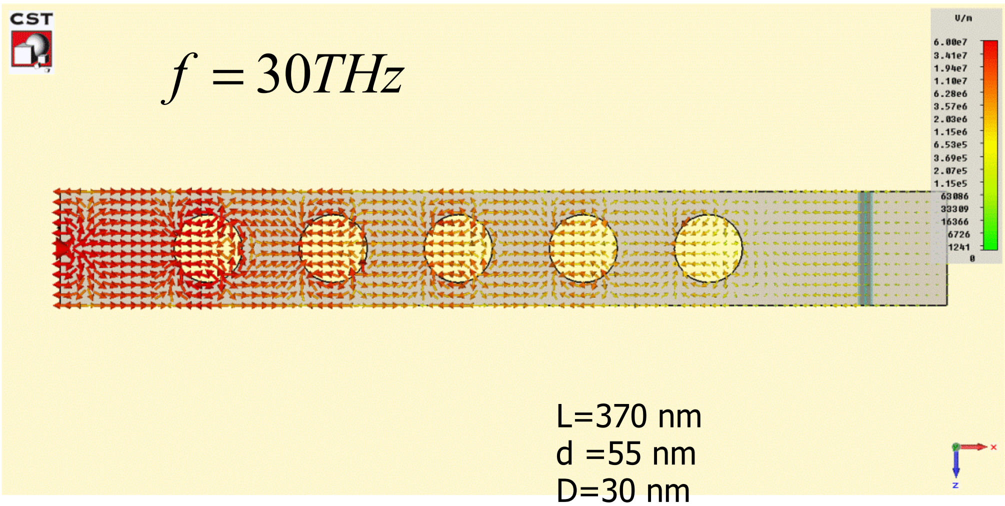


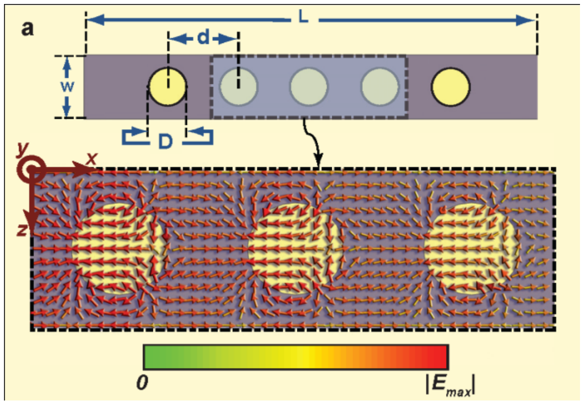
Region 1: $\sigma_{g,i} > 0$

$\mu_c = 150\ meV$

Region 2: $\sigma_{g,i} < 0$

$\mu_c = 65\ meV$





L=370 nm
d =55 nm
D=30 nm
w=55 nm